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THE *PAÑCAVIṂŚATIKĀ* IN ITS TWO RECENSIONS: A Study in the Reformation of a Medieval Sanskrit Mathematical Textbook

TAKAO HAYASHI

Doshisha University, Kyoto 602, Japan

INDIAN NATIONAL SCIENCE ACADEMY
BAHADUR SHAH ZAFAR MARG
NEW DELHI-110002

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INTRODUCTION

A small mathematical treatise in Sanskrit, the *Pañcaviṃśatikā*, together with Śambhunātha's vernacular commentary, *Gaṇitasāra*, is contained in the manuscript,

A: LD Institute, Ahmedabad, Acc. No. 5325.

The date of Śambhunātha is not known precisely, but he refers to Bhāskara II's *Līlāvatī* (A.D. 1150). In addition, he employs in examples two monetary units called *korī* and *dokaḍā* (28 dokaḍās = 1 korī). The former is most probably a copper coin minted by Sultan Muzaffar Shah III (A.D. 1562-72) [Sircar 1968, 23]. According to the colophon, the manuscript was copied by a Caturavijaya at Bhujanagara (modern Bhuje in Kaccha, about 300 km west of Ahmedabad) in Saṃ 1786 = A.D. 1730. Śambhunātha, therefore, seems to have flourished some time between A.D. 1562 and 1730.

The same work, together with a Gujarātī commentary, *Bālabodhānkavṛtti* is contained in the manuscript,

B: Oriental Library, Baroda, Acc. No. 5283.

According to the last verse and the colophon of the manuscript, the *Bālabodhānkavṛtti* was composed by a *mantrin* (a wise man or a minister?), Śambhudāsa by name, at Ahmedabad in Saṃ 1485 = A.D. 1428/29. It follows that the *Pañcaviṃśatikā* was written before A.D. 1429. But neither of the two manuscripts mentions the name of its author.

The manuscripts

A: LD Institute, Ahmedabad, Acc. No. 5325.

Title: *Gaṇitasāra*. Script: Devanāgarī, Ff. 1-5. Complete. 17 lines to a page. About 55 to 60 letters (*akṣaras*) to a line. Paper. The *Gaṇitasāra* ends at the middle of fol. 5a, and on the rest of the same page is written a *naṣṭajanmapatrī* ("a leaf for a lost horoscope"). On fol. 5b are written several rules concerning *horās* ("hours"), and a table of 28 (not 27) *nakṣatras* which lists the number of stars and their formation (with an illustration) of each *nakṣatra*.

B: Oriental Library, Baroda, Acc. No. 5283.

Title: *Gaṇitapañcaviṃśatikābālāvabodha*. Script: Devanāgarī. Ff. 1-10. Complete. 12 lines to a page. About 40 to 45 letters (*akṣaras*) to a line. Paper.

The languages of the commentaries

The language used in Śambhudāsa's *Bālabodhānkavṛtti* is basically Old Gujarātī, but it often comes very near to Sanskrit, and sometimes it may be regarded as almost

regular Sanskrit, although several phonetic peculiarities, probably due to the author's native tongue, are observed.

Śambhunātha's *Gaṇitasāra* is written throughout in a vernacular, which too is perhaps Old Gujarātī, but which is slightly different from that of Śambhudāsa.

Reconstruction of a common ancestor

The title of the work, *Pañcaviṃśatikā* ('[the work] of twenty-five [verses]), suggests that the work in its original form consisted of 25 verses. Manuscript A, however, contains 40 verses, among which 39 are numbered 1, 1-3, 1-6, 6-33, and 1; another unnumbered verse is embedded in the commentary on (A 22). Manuscript B, on the other hand, contains 27 numbered verses plus several that are unnumbered. Many verses, with or without variants, are naturally found in both manuscripts, but some are only in A, and others only in B. These facts indicate that Śambhunātha and Śambhudāsa separately commented on an enlarged recension of the *Pañcaviṃśatikā*, although it is not easy to differentiate their own additions from their predecessors'.

I have tried to reconstruct an ancestor (designated X) common to both recensions (A and B) according to the following working hypotheses (cf. the Concordance below).

1) Those verses which occur in only one of the two recensions are later additions.
   Verses found only in A:
   1', 2', 3', 3ab, 4, 5, 21cd, 22, 22U, 23, 24, 29, 30, 31, 32, 1°.
   Verses found only in B:
   9U, 19, 21U, 25, 26, 27.

2) The other verses, which occur in both recensions, existed in the ancestor.

The first three verses in the above list, (A 1'), (A 2'), and (A 3'), which are found only in A, are numbered separately from the rest in the manuscript. Moreover, Śambhunātha introduces the next stanza, (A 1) = (X 2), as "the *śloka* stanza of the original work" (mūlagraṃtha no ślokaḥ). This means that the three verses at least were added by Śambhunātha himself. (A 22U) occurs in the solution of an example under (A 22), and is not numbered in the manuscript. It is, therefore, most probably a quotation by him. (A 1°) occurs in the commentary on the last verse, (A 33), and is numbered "1" separately from the rest. So, it also appears to be an addition by him.

As for recension B, the last verse, (B 27), which is a colophon to the commentary, is no doubt an addition by Śambhudāsa himself. The two unnumbered verses, (B 9U) and (B 21U), which are a quarter verse and a half verse respectively, are also most probably additions by him.

Apparent exceptions to the second hypothesis are (A 3cd) ≃ (B 2U), and (A 21ab) ≃ (B 18U). Although each pair has the same content, I have excluded them for the following reasons. First, the sentences are not the same; (A 3) and (A 21) are written in

normal *āryā* and *śloka* meters respectively, while (B 2U) is not quite metrical and (B 18U) is poorly worded in *śloka*. Second, both (B 2U) and (B 18U) are unnumbered in recension B, and correspond to only part of a full verse, (A 3) and (A 21) respectively, of recension A. Moreover, (A 3) is identical with *Triśatikā* 1 = *Pātīgaṇita* 14 of Śrīdhara (of the eighth century A.D.). It is, therefore, highly probable that they are later additions. It may also be pointed out in this connection that (A 21) agrees with *Aparājitapṛcchā* 47.22 (in the twelfth century) and Ṭhakkura Pherū's *Gaṇitasāra* 3.45 (ca. A.D. 1315) in contents. See Translation below.

Thus, I have obtained twenty-four *ślokas* of the supposed ancestor; they are numbered 1-24 in the third column (designated X) of the Concordance given below. This is quite reasonable because we get exactly 25 *ślokas* by supposing another one for the lost colophon of the original *Pañcaviṃśatikā*. It is, therefore, very likely that these twenty-four verses constituted the original *Pañcaviṃśatikā* too, but it is sometimes difficult to determine the original purports of those verses precisely, since the two commentaries differ from each other sometimes in the expressions of the verses, and sometimes in their interpretations. We shall examine these differences next.

Differences between the two commentaries

I shall give below a list of the differences between the two commentaries with brief comments. See the Translation with Mathematical Commentary below for more information.

(X 1) Benediction.
A: *svīyasūtrair* (with my own versified rules).
B: *pañcasūtrair* (with five *sūtras*).

In recension B, the word *sūtra* is used in an unusual sense, i.e., "a section" of the text,and in fact Śambhudāsa divides the text into five sections.

Sūtra 1. Addition (1 ½ verses): (B 2)-(B 2U).
Sūtra 2. Subtraction (1 verse): (B 3).
Sūtra 3. Multiplication (6 verses): (B 4)-(B 8U).
Sūtra 4. Division (1 ¼ verses): (B 9)-(B 9U).
Sūtra 5. The rest (19 verses): (B 10)-(B 27).

It is, however, noteworthy that nothing in the ancestor reconstructed above suggests such a division into sections. Recension A has no explicit division into sections, as is usual with small mathematical works like this.

(X 2) Sum of the natural series.
A: Śambhunātha reads only two methods in the verses.
B: Śambhudāsa reads four methods.

As the verse actually contains at least three methods, Śambhunātha seems to have missed at least one method. Śambhudāsa, on the other hand, reads four methods, but his reading of the verse contains an irregular hiatus.

(X 3) Difference of the natural series.

A: According to Śambhunātha, *vyavakalita* ("difference") is,

$S_n - n = S_{n-1}$

B: According to Śambhudāsa, it is,

$V_{n.m} = S_n - S_m.$

Śambhunātha's interpretation of *vyavakalita* is unique, while Śambhudāsa agrees with Śrīdhara (Tr and PG) and Ṭhakkura Pherū (GS).

(X 5) *Kapāṭasandhi.*

(X 6) *Gomūtrikā.*

(X 7) *Tatstha.*

(X 8) *Hīnādhikabhāga* of *khaṇḍaguṇana.*
These are the names of multiplication methods, but the nomenclature is hopelessly confused in the two commentaries.

(X 9) Multiplication of zero and unity

A: Śambhunātha reads only two rules in the verse.

B: Śambhudāsa reads three rules.

Śambhudāsa's second rule, which Śambhunātha misses, is a rather unusual one, prescribing the writing-down of the zero symbol at a certain step of the multiplication procedure.

(X 14) Fractions

A: Multiplication and division of fractions of the ordinary type, b/a.

B: Multiplication of numbers whose fractional portions are expressed according to a place-value system, $p;q$.

Recension A is traditional.

(X 15) Rule of three

A: (the first, the middle, the last).

B: (the last, the middle, the first).

The order of the three terms in the terminology of recension B is unusual. Recension A is traditional.

(X 16) Investments

A: One method.

B: Two methods, according to Śambhudāsa.

It is difficult to see Śambhudāsa's second method even in his own wording of the verse.

(X 17) Second rule, which is meant for the purification of gold.
A: Rule for the purity.
B: Rule for the weight.
Perhaps the rule itself is meant for the weight as in B, but Śambhudāsa wrongly applies the rule to an example of simple mixture of gold.

(X 23) Second rule, which is meant for the volume of a cone-like heap of grain.
A: Śambhunātha wrongly takes the rule to give a formula for the volume of a sphere.
B: Śambhudāsa's interpretation is correct.

(X 24) Telling time from a shadow
A: Correct and exact expression of a rule.
B: Ambiguous expression of the rule.
(A 1°) in A and (B 26) in B give a table of twelve noon-shadows, which are to be employed in (X 24), but the numerical values in the two tables differ from each other. It is of special interest that the values given in (A 1°) do not satisfy a simple linear zig-zag function. Those of (B 26), on the other hand, satisfy as usual a linear zig-zag function. The maximum and minimum values of both functions are six and zero respectively, but in A they take place at Sagittarius and Gemini respectively; in other words, the entire graph of A has been shifted to the left by one zodiacal sign.

THE ORIGINAL *Pañcaviṃśatikā* AND THE TWO RECENSIONS

Although Śambhunātha does not always understand the verses correctly (X 3 and 23), recension A itself seems to preserve more traditional elements than recension B, which contains several new elemetns (X 1, 14, and 15) that do not completely tally with traditional Indian mathematics. Śambhudāsa employs in his commentary a linear standard of non-Sanskrit origin, namely, *gaja*, which called *gaz*, was in vogue under the Mughul Empire. Abu-l Fazl (ca. A.D. 1600) has recorded seven types of *gaz* [Srinivasan 1979, 15]. The inverse order of terminology for the rule of three (X 15), adopted in recension B, may also be explained by the right-to-left order of Arabic writing. At the same time, however, it should be noted that Śambhudāsa has borrowed many examples from Śrīdhara's works, *Triśatikā* and (or) *Pāṭīgaṇita*.

The reconstructed ancestor (X) begins with the sum of the natural series *(saṅkalita)*. This is in the style of Śrīdhara (both in the *Triśatikā* and the *Pāṭīgaṇita*) and of Ṭhakkura Pherū (in the *Gaṇitasāra*). Śambhunātha supplies a verse for the addition and subtraction of numbers expressed in the decimal place-value system, following probably the style of Bhāskara II's *Līlāvatī*, which is in fact the only work Śambhunātha mentions by name. See under (A 28) = (X 23).

Medieval Indian mathematics consisted of two major fields, namely, *pāṭī* and *bījagaṇita*. The original *Pañcaviṃśatikā* may be classified as a book of *pāṭī*, which

consists mainly of arithmetical operations (*parikarman*) such as addition, subtraction, etc. and eight kinds of "practical mathematics" (*vyavahāra*), and which does not employ algebraic symbolism (*bīja*). The ancestor actually contains most of these topics, but not all of them. It misses "the practical mathematics" of "piling [of bricks]" (*citi*) and "sawing [of timbers]" (*krakaca*), and is very poor in "the practical mathematics" of "series" (*śreḍhī*) and "plane figure" (*kṣetra*). Thus, the ancestor is incomplete as a book of *pāṭī*. This was probably the case also with the original *Pañcaviṃśatikā*, and recension A was perhaps meant to be a more comprehensive book of *pāṭī*, although it still lacks the section on "the practical mathematics of piling", and rules based on the Pythagorian Theorem.

From the viewpoint of the history of Indian mathematics, the importance of our text lies in its historical expansion and reformation rather than in its mathematical contents, as it throws new light upon the history of reformation of other Sanskrit mathematical treatises. For example, the title of our text, *Pañcavimśatikā*, reminds us of the *Ganitapañcavimśī* "Mathematics in twenty-five *ślokas*" "ascribed to Srīdhara (in the eighth century). Although there seems to be no direct relation between the two works, both claim to be a work consisting of twenty-five verses, and both contain more than that in their present form; the number of verses of the latter is estimated at nearly ninety from its unique, incomplete manuscript. This seems to me to suggest that, just like our text, the *Ganitapañcavimśī* also consisted of twenty-five verses in its original form, and was expanded to its present form by the hands of a later interpolator or interpolators. This conjecture is supported by the fact that eight and a half verses of the present *Gaṇitapañcaviṃśi* are found verbatim in Bhāskara's *Lilāvatī* (A.D. 1150).

Gaṇitapañcaviṃśī	*Lilāvatī*
Rule 26	204 : 3/2 Indravajrā stanzas
Ex. 14	205 : 1 Śloka
Rule 27	135 : 1 Āryā
Rule 28	136 : 1 Āryā
Ex. 15	137 : 1 Śloka
Rule 31ab	234 : ½ Vasantatilakā
Ex. 19	235 : 1 Indravajrā
Rule 31cd	236 : ½ Vasantatilakā
Ex. 20	237 : 1 Upajāti

Other works having similar titles

Tejasiṃha, head of the Lunkāgaccha, wrote in the 17th century the *Iṣṭānkapañcavimśatikā* ("[A book of] twenty-five [stanzas/rules] on arbitrarily chosen numbers", a manuscript of which is preserved in the LD Institute (Acc. No. 903). It consists of 25 verses (14 *śārdūlavikriḍita* stanzas and 11 *vasantatilakā* stanzas) for mathematical rules and one additional verse (in the *upajāti* meter) for the ending.

All the rules deal with a single type of mathematical game, in which any optional number, when modified by a series of arithmetical operations, turns out to be the number determined in advance. The ninth stanza, for example, reads thus:

Any optional number is multiplied by 265, increased by unity, multiplied by 500, and divided by 265. The remainder [of the division] is again multiplied by 2. When the years equal in number to the result have elapsed from the Final Liberation (*nirvāna*) of Mahāvīra, King Vikrama was born to the earth.

The arithmetical computations have been so arranged here that number 470 is obtained irrespective of the number chosen by a partner.

The author, Tejasiṃha, is said to have died in A.D. 1686 [Jain & Agrawal 1988, 22]. Photocopies of the manuscript have appeared at the close of *Arhat Vacana,* Vol. 1, nos. 3-4, 1989.

It is probably the same Tejasiṃha that wrote the *Gaṇitapañcaviṃśatikā* (Ms: LD Institute, Acc. No. 3665/5, dated Saṃ 1870 = A.D. 1813). It is a small collection of 25 mathematical problems, each problem being questioned in one *drutavilambita* stanza. Most of the problems are of the type:

$$x - \frac{x}{2} - \frac{x}{3} - \frac{x}{42} - \frac{x}{8} = 18 \text{ (stanza 25)}.$$

The answer 1008 to this problem is a sacred number for the Jainas. Another stanza, numbered 26, in the same meter is appended for the ending.

These two works, devoted for particular topics, may be regarded as a kind of monograph, which is a hitherto neglected field of study in Indian mathematical literature.

Concordance of the Sanskrit verses commented on by *Śambhunātha* (A) and *Sambhudāsa* (B) *with their ancestor (X) reconstructed*

In manuscript A, there are four verses numbered 1, two 2, two 3, and two 6. In order to differentiate them, the marks, '', ', and °, have been attached to the numbers by me. The letter U attached to a verse-number (*N*) indicates that the verse in unnumbered and located between verses *N* and *N* + 1 in the manuscripts.

A	B	X	Contents
1''	1	1	Benediction to Mahādeva. Calls the work *Pañcaviṃśatikā*.
1'	—	—	Names of 18 decimal places.
2'	—	—	(Continued)
3'	—	—	Addition & subtraction in the decimal place-value system, with a rule for zero.
1	2	2	Sum of the natural series.
2	3	3	Difference between the sums of two natural series.
3ab	—	—	Sum of the natural series (S_n).

3cd	2U	—	Integral portion of $\sqrt{2S_n} = n$. (A3) = Tr 1 = PG 14
4	—	—	Arithmetical progressions.
5	—	—	Geometrical progressions.
6	4	4	Multiplication: Enumerates the names of 4 methods.
6′	5	5	*Kapāṭasandhi*.
7	6	6	*Gomūtrikā*.
8	7	7	*Tatstha*.
9	8	8	*Khaṇḍa*.
10	8U	9	By zero and unity.
11	9	10	Division.
—	9U	—	Zero for a quotient. (B 9U) is a quarter verse.
12	10	11	Square and cube.
13	11	12	Square-root.
14	12	13	(Continued)
15	13	14	Multiplication and division of fractions. The rules of A and B are different.
16	14	15	Rule of three and inverse rule of three.
17	15	16	Investments.
18	16	17	Mixture of gold (its purity).
19	17	18	Areas of a square and an oblong. Area of a trapezoid.
20	18	19	Areas of a triangle and a circle.
21ab	18U	—	Circumference of a circle.
21cd	—	—	Area of a circle.
22ab	—	—	Area of a circle.
22cd	—	—	Area of a circle.
22U	—	—	Approximation to the square root.
23ab	—	—	Area of a circle (identical with A 22 ab).
23cdef	—	—	Area of a circle.
24	—	—	Area of a segment of a circle.
—	19	—	Heron's formula for the areas of a trilateral and a (cyclic) quadrilateral.
25	20	20	Area of a segment of a circle. Area of "a rim", or of any other irregular figure.
26	21	21	Volume of a rectangular solid.
—	21U	—	Arithmetical mean of different lengths. (B 21U) is a half verse.
27	22	22	Volume of a cylinder.
28	23	23	Volume of a sphere. Volume of a cone-like heap of grain.
29	—	—	Grain heaped up on the ground.
30	—	—	(Continued)
31	—	—	(Continued)
32	—	—	Work for sawing timber.

33	24	24	Telling time from a shadow.
1°	—	—	Length of the noon-shadow.
—	25	—	Length of daylight.
—	26	—	Length of the moon-shadow.
—	27	—	Colophon of the commentary. Gives the name of the author (Śambhudāsa) and the title (*Bālabodhānkavṛtti*) of the commentary.
[—	—	25	Colophon of the *Pañcaviṃśatikā* (?)]

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SANSKRIT TEXT OF THE *PAÑCAVIṂŚATIKĀ*

Two recensions (A and B) and their common ancestor (X) reconstructed.

(X 1)>(A 1″, B1)

mahādevaṃ praṇamyādau bālānāṃ buddhivṛddhaye[1]/
svīyasūtrair[2] ahaṃ vakṣye pañcaviṃśatikām imām/ /
[1]-vṛddhāye in B (corr.i.m.). [2]pañcasūtrair in B.

(A 1')

ekaṃ daśa śataṃ tasmāt sahasram ayutaṃ tathā/
lakṣaṃ ca prayutaṃ koṭir arbudaṃ cābjakaṃ tataḥ/ /

(A 2')

kharvanikharvapadmaṃ ca śa[nkur] jaladhir eva ca/
Antyaṃ madhyaṃ parārdhaṃ ca kramād ākhyā daśottarāḥ/ /

(A 3')

yathā sthānakam aṅkānāṃ yutir viyutir āditaḥ/
khenonāḍhyaḥ sa eva sa na[1] pāte [']ṃtyāṃkasaṃgraha[ḥ]/ /
[1]saḥ/da in ms.

X 2)>(A 1, B 2)

saikādyapadaghātārdhaṃ bhaved[1] ekottareṇa tat[2]/
padavargādyayogārdhaṃ[3] saikādyārdhavadhe[4] phalam/ /
[1]tathā in B. [2]ca in B. [3]-yuktārdhaṃ in B. [4]saikādyardhavadheḥ in A.

(B 2U)

saṃkalitadviguṇān mūlasamo[1] gaccha[ḥ]/ /
[1]mūlūṃ samaṃ is ms.

(X 3)>(A 2, B3)

saṃkalitotpannadyumnād[1] vyayaṃ tyaktvā[2] dhanaṃ bhavet/
taddhanaṃ[3] vyavakalitaṃ munibhiḥ purā/ /
[1]saṃkalitonnatadrarmmā in A, -dyumnāta in B. [2]kṛtvā in B.
[3]tat- in B.

(A 3)

saikapadāhatapadadalam ekādicayena bhavati saṃkalitam/
dyiguṇīkṛtasaṃkalitān[1] mūlaṃ gaccho [']vaśiṣṭasamaḥ/ /
[1]-saṃkalitāt in ms.

(A4)

vyekapadaghnacayaḥ sādyo[1] [']ntya[ṃ] svavaktrayutād[2] dalam/
madhya[ṃ] svapadanighnaṃ tat sarvasvaṃ jāyate caye/ /
[1]-cayogrāhyo is ms. [2]-yutāṃ in ms.

(A 5)

oje gacche nireke syād guṇaḥ same [']rddhite kṛtiḥ/
gacchāntaṃ vyastam antyānkāc cayād guṇāh kṛtiḥ[1] phalam/
vyekaṃ vyekacayacchinnam ādighnaṃ syād guṇottare/ /
[1]kṛtaṃ in ms.

(X 4)>(A 6, B4)

dvidhā kapāṭasandhiś ca tathā gomūtrikā dvidhā|
tastho dvidhā punaḥ proktas tathā khaṇḍas[1] tridhā smṛtaḥ[2]| |
[1]khaṇḍa-in B. [2]mataḥ in A.

(X 5)>(A 6', B5)

praśnopari nyasen mūlyaṃ mūlyena[1] guṇayet kramāt|
anulomavilomābhyāṃ kapāṭākhyam dvidhā bhavet| |
[1]mūlaṃ maulyena in A, mūlaṃ mūlona in B.

(X 6)>(A 7, B 6)

praśnād adho nyasen mūlyaṃ[1] guṇayet sakalaṃ mithaḥ[2]|
anulomavilomābhyāṃ gomūtrākhyaṃ dvidhā bhavet| |
[1]mūlya- in B. [2]tataḥ in A, guṇayeś| |śrīlaṃ mithaḥ in B.

(X 7)>(A 8, B 7)

ekaikaguṇanād rāśeḥ śīrṣabhedo[1] nigadyate[2]|
pṛṣṭhabheda[ḥ][3] punaḥ proktaḥ tastho [']pi dvividhaḥ[4] smṛtaḥ[5]| |
[1]śīrṣā- in B (corr.i.m.). [2]nigadyataḥ in B. [3]pṛṣṭabhede in A,
koṣṭābheda in B (corr.i.m.). [4]dvividheḥ in B (corr.i.m).
[5]tasthepidvividhāpunaḥ in A.

(X 8)>(A 9, B 8)

kvacid[1] rūpavibhāgaś ca sthānabhāgaḥ kvacid bhavet|
kvacid dhīnādhiko[2] bhāgaḥ khaṇḍo [']pi trividhaḥ[3] smṛtaḥ| |
[1]kvacit in B. [2]kvacit hīnādhiko in B. [3]dvividhaḥ in A.

(X 9)>(A10, B 8U)

śūnyena guṇitaṃ[1] śūnyaṃ śūnyam agre[2] niyojayet|
tad evaikena[3] guṇitaṃ bhaved evaṃ[4] hi sarvataḥ| |
[1]guṇe taṃ in A. [2]śūnyamārge in A.
[3]tadvadekena in A, tadaivaikena in B. [4] eva in A.

(X 10)>(A 11, B9)

praśnād adho haraṃ nyasya praśnaṃ chitvā[1] hareṇa ca|
bhāgo hāryaḥ kramān nūnaṃ bhāgāhāravidhiḥ smṛtaḥ| |
[1]bitvā in A.

(B 9U)

bhāgo nāsti labdhaṃ śūnyaṃ| |

(X 11)>(A 12, B 10)

tulyarāśidvayor[1] ghāte vargo bhavati kevalam|
padatrayāṇāṃ[2] tulyānāṃ vadhe nūnaṃ ghano bhavet| |
[1]-rāśyodvayor in A. [2]-triyāṇāṃ in B.

(X 12)>(A 13, B 11)

vargaṃ saṃtyajya viṣamād dviguṇena[1] paraṃ ghajet|
labdhaṃ niveśayet[2] paṅktyāṃ tadvargaṃ paratas tyajet[3]|
[1] viṣamāt dviduṇana in B. [2]viniveśayet in A. [3]pariśodhayet in B.

(X 13)>(A 14, B 12)

pūrvavad[1] dviguṇīkṛtya[2] tad utsārya[3] paraṃ bhajet[4]|
evaṃ pare[5] [']pi kartavyaṃ dalayed[6] dviguṇīkṛtam| |
[1]pūrvava- in A, pūrvavat in B. [2]dviguṇaṃkṛtya in A.
[3]ucchārtha in B. [4]bhavet in A. [5]paro in B. [6]dalaye in A,
dalayet in B.

(X 14)>(A 15)

aṃśahati[ś] chedaghātabhaktā bhinne guṇe phalam|
paravarttyāṃśakacchedau guṇavad bhāgake phalam| |

>(B 13)

rūpaṃ rūpaiḥ samaṃ guṇyaṃ chedāṃśāv[1] api taiḥ[2] sama[m]|
paraspara [ṃ] tadaṃśau ca bhinnotpannaṃ phalaṃ labhet| |
[1]cchidāṃśāv in ms. [2]teḥ in ms.

(X 15)> (A 16)

madhyāntyayor vadhaḥ pūrvam ādibhāgas trirāśike|
madhyādyayor vadho[1] vyaste prāntyabhaktaḥ phalaṃ bhavet| |
[1]madhyādayor vyadho in ms.

>(B 14)

madhyādyayor vadhaḥ pūrvam antyabhāgas trirāśike|
madhyāntyayor vadho vyaste ādibhāga [ḥ] phalaṃ labhet| |

(X 16)>(A 17)
nānāprakṣeparūpāṇāṃ utpannena phalena te|
guṇyāḥ svayogabhaktās te khaṇḍāni syuḥ pṛthak pṛthak| |
>(B 15)
nānāprakṣeparūpāṇām utpannena vadhaḥ kramāt|
yutibhāgā[d] dyayau[1] labdhī phalaṃ syāt tat pṛthak[2] pṛthak| |
[1]dvayā in ms. [2]prathak in ms.

(X 17)>(A 18)
suvarṇavarṇaghātyaikyaṃ svarṇayogena bhājayet|
labdhāṅke varṇakā jñeyā svarṇoddhṛte[1] sa varṇakaḥ||
[1]varṇoddhṛtte in ms.
>(B 16)
nānāsuvarṇavarṇānām aikyaṃ svarṇena[1] bhājyate|
labdhāṅke varṇikā jñeyā[2] suvarṇaṃ varṇikāhṛtam| |
[1]svarṇyena in ms. [2]carṇikā jñaiyā in ms.

(X 18)>(A 19, B 17)
caturasre[1] same dīrghe bhujakoṭivadhaḥ phalam|
vasudhāmukhayogārdhaṃ lambaghnaṃ viṣame phalam| |
[1]caturaśre in B.

(X 19)>(A 20, B 18)
vasudhārdhaṃ lambanighnaṃ[1] phalam kṣetre[2] trikoṇake|
vyāsasya turyabhāgena[3] vadho[4] vṛtteś ca vartule[5]| |
[1]vasudhārdhalambaghātaḥ in B. [2]kṣetri in B. [3]-bhāgana in B.
[4]vadhoḥ in A. [5]vartuleḥ in A.
(A 21)
vyāsas triguṇito[1] vṛttir vyāsaṣaḍbhāgasaṃyuthaḥ[2]|
vyāsārdhaṃ ca paridhyardham anyonyaṃ paritāḍayet| |
[1]vyāsattriguṅitā in ms. [2]-yuto in ms.
(B 18 U)
vyāsasya triguṇā paridhiḥ ṣaḍbhāgasamanvitā[1]||
[1]ṣaṭ- in ms.
(A 22)
vyāsavarge phalaṃ sthūlaṃ pañcamāṃśanipātanāt|
vyāsārdhavargavargaṃ ca daśaghnaṃ mūlataḥ phalam||
(A 22U)
mūlāvaśeṣakaṃ saikaṃ ṣaṣṭighnaṃ vikalānvitam|
dviguṇena dviyuktena[1] mūlenāptaṃ sphuṭaṃ bhavet| |
[1]dviguṇā- in ms.
(A 23)
vyāsavarge phalaṃ jñeyaṃ pañcamāṃśanipātanāt|
vyāsavargasya yo rāśir hīnas turyeṇa kārayet|
svadhṛter bhāgasaṃyuktaṃ phalaṃ jñeyam athāpi vā| |

(A 24)

jyāśaraikyadalaṃ bāṇanighnaṃ tadvargadigghateḥ|
navabhaktasya mūlaṃ taddhanurbāṇaphalaṃ smṛtam| |

(B 19)

(bhujaikyārdhaṃ[1] caturdhā ca bhujahīnaṃ ca tadvadhāt[2]|
mūlaṃ phalaṃ bhaven nūnaṃ tryasre[2] vāpi caturmukhe| |
[1]bhujekyārdhaṃ in ms. [2]taghāt in ms. [3]tisre in ms.

(X 20)>(A 25, B 20)

jyāśaraikyadaleṣughnaṃ[1] bhāgāṣṭādaśasaṃyutam[2]|
nemyākṛtau ca viṣame vyāsadīrghavadhaḥ[3] phalam| |
[1]yyāśaraikyadaleyyughnaṃ in B. [2]sāgāṣṭā- in A, bhāgāṣṭaḥ daśa- in B. [3]vyosaṃbavadhaḥ in B.

(X 21)>(A 26, B 21)

khāte kāṣṭhe[1] ca[2] pāṣāṇe koṣṭhāgāre[3] citau tathā|
vyāsadīrghavadhas[4] tatra piṇḍaghātaḥ same[5] phalam| |
[1]kāṣṭe in A. [2]B om. ca. [3]koṣṭhāre in B. [4]-dīrdhavadhās in B. [5]piṇḍaghātavadhaḥ in A, piṇḍaghāte samaṃ in B.

(B 21U) tathā vakram|

viṣamaṃ yadi cet [tena] viṣamayuti[bhāgakaḥ][1]| |
[1]tathā ca| kraṃ biṣamaṃ yade ceta viṣamayuti caitataḥ kuryāt in ms.

(X 22)>(A 27, B 22)

kāṣṭhapāṣāṇayo[ḥ] stambhe kūpe ca sadṛśaṃ[2] phalam|
vṛttakṣetre[3] phalaṃ tatra tena piṇḍavadhaḥ[4] phalam| |
[1]kāṣṭa- in A. [2]kūpe va sadraśaṃ in B. [3]-kṣetra- in B.
[4]piṇḍasamaṃ in A (cf. note 5 for (X 21)), -vādhaḥ in B.

(X 23)>(A 28, B 23)

golavyāsaghanārdhaṃ[1] ca bhāgāṣṭādaśasaṃyutam|
vṛtteḥ ṣaḍaṃśas tadvarga[2] udayena hataḥ phalam| |
[1]-vyāsanārdhaṃ in B. [2]vṛttaṣaḍaṃśavargas tad- in B. [3]hattaḥ in B.

(A 29)

aṇuṣv ekādaśāṃśaḥ syād vedho [']naṇuṣu diglavaḥ|
śūkidhānye ['n]kabhāgaḥ syāt paridheś ca ṣaḍaṃśakaḥ| |

(A 30)

vargito vedhaguṇito ghanahastamitir bhavet|
madhyāntarbāhyaparidhir dvyabdhisatryaṃśarūpakaiḥ| |

(A 31)

guṇitā svīyahāreṇāptaṃ phalaṃ bhittimadhyathaḥ|
antarbāhyasthakoṇeṣu phalaṃ ghanakarāḥ smṛtāḥ| |

(A 32)

bāhukoṭivadhārddhaṃ ca tryasrakṣetre phalaṃ smṛtam|
mūlāgrapiṇḍayogārddhaṃ dairghyaghnaṃ dārujaṃ phalam| |

(X 24)>(A 33, B 24)

chāyā saptayutā kāryā madhyacchāyāṃ[1] tatas tyajet।
dinārddhaṃ saptabhir guṇyaṃ tenāptaṃ ghaṭikādikam[2]। ।
[1]-cchāyā in A. [2]Second line in B: dinārdhāt saptabhir guṇyād bhāgān nāḍyo gatā [ḥ] sthitāḥ ("tatas...guṇyā" in margin).

(A 1°)

dvyekakhaikaikasārddhadvivipādvedāś[1] ca paṃcakam।
ṣaṭ [ca] paṃca vipātpaṃca trayaṃ sārddhaṃ ca madhyabhā। ।
[1]dvyekakhabhaikasārddhaddhivipādvedāś in ms.

(B 25)

makarādidinās trighnāḥ[1] khatripañcaikasaṃyuktā[ḥ]।
ṣaṣ[ṭ] er bhāgāt kalā labdhi[ḥ] karkād rātris tathaiva hi। ।
[1]-dināḥ।tri- in ms.

(B 26)

tridvyekakhendupakṣāgniyugeṣuṣaṭśarā[1] yugā[ḥ]।
kramān meṣādisaṃkrāntau madhyapādāḥ prakīrttitāḥ[2]। ।
[1]-yugeyyu- for yugeṣu in ms. [2]-pādapra- in ms.

(Colophon of A)

iti śrīśambhunāthaviracitaṃ gaṇitasāraṃ samāptam।। saṃ 1786 varṣe māha vadi 13 dine likhitam। ga[ṇi]। caturavijayena।। śrībhujanagare।। svavācanārtham।। ।।śrīr astu।। kalyāṇam astu।। śreyo [']stu।। iti bhadraṃ syāl lekhakapāṭhakayoś ca।।

(B 27)

bāṇāhivedacandrābde[1] [']hammadāvādapattane।
bālabodhāṅkavṛttiś[2] ca śambhudāsena[3] nirmitā।।
[1]vāṇāhideva- in ms. [2]-pṛṣṭiś in ms. [3]śuṃbhu- in ms.

(Colophon of B)

iti mantriśambhudāsaviracitā[1] pañcaviṃśatikābālabodhāṅkavṛttiḥ samāptā[2]।।cha।। ।।cha।।
[1]maṃtraśaṃbhudāsaviricitā in ms. [2]samāptaḥ in ms.

A 1'-2'. Names of 18 decimal places

Eka (10^0), daśa (10^1), śata (10^2), sahasra, ayuta, lakṣa, prayuta, koṭi, arbuda, abjaka (10^9), kharva, nikharva, padma (10^{12}), śaṅku, jaladhi, antya, madhya, and parārdha (10^{17}). [These are] named in order according to the decimal system.

These names, except for *padma*, were very popular in Hindu (as against Jaina) mathematical works since the eighth century A.D. The number of places, 18, was also fixed in them. The popular name for the power 10^{12} is *mahāpadma* (a big lotus) or its synonyms (such as *mahābja, mahāpaṅkaja, mahāsaritpati*, etc.); and *padma* (a lotus) without *mahā* (big) is usually taken to indicate the same power as *abja* (a lotus), i.e., 10^9. In the present stanzas and Śambhunātha's commentary, however, *padma* is regarded as different from *abja (ka)*.

Śambhunātha refers to the decimal places greater than *parārdha;* he repeats *daśa, śata, sahasra,* etc. together with *parārdha*. His example is: "eka sahasra eka śo igyāra parārdha" (one thousand one hundred and eleven *parārdha*) for 1111×10^{17}.

Cf. Abh 2.2; Tr 2'-3'; PG 7-8; GP 8-9; GT 2-3; L 10-11; GS 1.12-14; GK Prakīrṇaka 2-3; GM 12-14. For a Jaina table see GSS 1.63-68.

A 3'. Addition and subtraction of numbers expressed in the decimal place-value system

Beginning with the first [numeral], the sum or difference of numerals [is made] according to the places. That [numeral], when decreased or increased by zero, is itself. When there is no subtraction (i.e., when the subtrahend is greater than the minuend), the sum of the last [adjacent] (i.e., next] numeral [and the minuend is taken to be the new minuend].

Ex. 1. Addition	Ex. 2. Subtraction
1 5 6	1 5 6 4 5
1 4	4 7 5 6
2 2 6	"These are subtracted."
1 1 3 5	1 0 8 8 9
1 5 3 1	

Cf. GP 10; L 12; GK Prakīrṇaka 12; GM 14.

X 2. Sum of the natural series ([saṃkalita]).

Half of the product of the first [value] increased by unity and the number of terms will be that [sum which is obtained] by increasing one by one. Half of the sum of the square of the number of terms and the first [value is also the sum]. Multiplying half of the first [or the first] increased by unity [by the other value, one obtains] the [same] result.

Def. $S_n = 1 + 2 + \ldots + n$.

$$Sn = \frac{n(n+1)}{2}.$$

$$Sn = \frac{n^2 + n}{2}.$$

$$Sn = \begin{cases} (n + 1) \cdot \dfrac{n}{2} \text{ (if } n \text{ is even).} \\ \dfrac{n + 1}{2} \cdot n \text{ (if } n \text{ is odd).} \end{cases}$$

Note that the word *ādya* (lit. "the first") in this rule is used in an unusual sense, i.e., "the number of terms", or *n*, which is also called *pada* as usual in the same verse.

A. Śambhunāta explains the first two methods only.
Ex. $n = 10$.

$$S_{10} = \frac{10\,(10+1)}{2} = 55. \qquad S_{10} = \frac{10^2 + 10}{2} = 55.$$

B. Śambhudāsa regards the definition itself as a method for computing *Sn*, reading the second quarter (*pāda*), of the verse, not as "bhaved ekottareṇa tat" (in ms. A), but as "tathā ekottareṇa ca" ([the same result is obtained] also by increasing one by one), which contains irregular hiatus.

Ex. $n = 10$.

$$S_{10} = \frac{10\,(10 + 1)}{2} = 55. \qquad S_{10} = 1 + 2 + \ldots + 10 = 55.$$

$$S_{10} = \frac{10^2 + 10}{2} = 55. \qquad S_{10}\,(10 + 1)\,\frac{10}{2} = 55.$$

For $n = 20, 30, \ldots, 100$, Śambhudāsa gives only the results: $S_{20} = 210$, $S_{30} = 465$, ..., $S_{100} = 5050$. This example is identical with Tr Ex. 1 = PG Ex. 1.

Cf. BSS 12.19; Tr. 1-2; PG 14-15; SS 13.21; L 117; GS 1.16-18; GK Śreḍhī 11 (I, p. 114); GM 155.

B 2U. Supplement to (X 2).

The number of terms is equal to [the integer part of] the square-root of twice the sum.

Integer part of $\sqrt{2Sn} = n$.

(B 2U) = (A 3cd) in contents. Cf. GS 1.20.

X 3. *Difference of the sums of two natural series (vyavakalita)*

Having subtracted the expense (*vyaya*) from the property (*dyumna*) produced by addition (*samkalita*), there will be property (*dhana*). This property has been called the difference (*vyavakalita*) by ancient sages.

A. Śambhunātha regards the verse as prescribing the rule:
$Sn - n = S_{n-1}$.
Ex. $n = 10$. $S_{10} - 10 = 55 - 10 = 45 = S_9$.

B. Śambhudāsa, like Śrīdhara and Pherū, regards the verse as giving the difference of the sums of two natural series (S_n and S_m):

$$V_{n,m} = S_n - S_m.$$

For S_n and S_m see (X2) above.
Ex. $S_{100} - S_{10} = 4995$, $S_{100} - S_{20} = 4840$, $S_{100} - S_{30} = 4585$, ..., $S_{100} - S_{90} = 955$, $S_{100} - S_{100} = 0$.

In the ms. these values are tabulated, and the last result, the zero (0), is expressed as "0000". This example is identical with Tr Ex. 2 = PG Ex. 2.

Cf. Tr 3-4; PG 16-17; GS 1.21-22.

A 3. Sum of the natural series

Half of the product of the number of terms and the same increased by unity is the sum by means of the first term and the common increase which are [both] unity. The [integer part of the] square-root of twice the sum, which is equal to the remainder, is the number of terms.

$$S_n = \frac{n\,(n+1)}{2}.$$

Integer part of $\sqrt{2 S_n}$ = its remainder = n,
(since $2S_n = n^2 + n$).

Ex. $n = 24$. $S_{24} = 300$. $2S_{24} = 600 = 24^2 + 24$.

The meter of (A 3) is *āryā*. Verse (A 3) is identical with Tr 1 = PG 14.

A 4. Arithmetical progressions

The common increase, multiplied by the number of terms less one and increased by the first term, is the last term. Half of the sum of [the result and] the first term is the middle (or the mean) term. This, when multiplied by the number of terms, becomes the total sum in piling (i.e., of an arithmetical progression).

$$a_n = a + (n - 1)\, d,\ \overline{m} = (a + a_n)/2,\ S_n = \overline{n\overline{m}}$$

where a, d, and n are the first term (*ādya*/*vaktra*), the common difference (*caya*), and the number of terms (*pada*) of an arithmetical progression, and a_n, $\overline{m}$, and S_n are the last term (*antya*), the middle term (*madhya*) or the arithmetical mean, and the total sum (*sarvasva*) of the series.

Ex. On the first Dvādaśī festival four Brāmaṇas are invited, and on the succeeding occasions of the same festival the number of Brāmaṇas invited is increased each time by three. How many are invited in total for twenty-five occasions of the Dvādaśī festival?

Given, $a = 4$, $d = 3$, $n = 25$. $a_{25} = 76$, $\overline{m} = 40$; $S_{25} = 1000$.

Cf. Abh 2.19; BSS 12.17; Tr 39; PG 85,95; GSS 2.64; MS 15.47; L 121; GS 3.26; GK Śreḍhī 1; GM 161.

A 5. Geometrical progressions

If the number of terms is odd, then [it is] decreased by one and there will be [the designation] "multiplication"; if even, then halved and "squaring". [This procedure is repeated until] the number of terms ends (i.e., is exhausted). In the inverse order from the last figure, which is the common increase (i.e., the multiplier), multiplication [by the common increase] and squaring [are made according to the designations]. The result [thus obtained], increased by one, divided by the common increase less one, and multiplied by the first term, will be [the sum of the geometrical series] of which the succeeding terms are multiples [of the preceding ones].

$$S_n = a. \frac{r^n - 1}{r - 1} \text{ (A 5ef)},$$

where a, r, and n are the first term, the common ratio, and the number of terms of a geometrical progression. (A 5abcd) gives an algorithm for computing r^n.

Ex. $a = 2$, $r = 3$, $n = 7$. (m = multiplication, s = squaring.)

$n = 7$
↓ m 2187 (= 729.3) = $3^7 = r^n$.
6 ↑
↓ s 729 (= 27^2)
3 ↑
↓ m 27 (= 9.3)
2 ↑
↓ s 9 (= 3^2)
1 ↑
↓ m 3 = r.
0

$$S_7 = 2. \frac{2187 - 1}{3 - 1} = 2186.$$

Cf. *Chandaḥsūtra* 8.32; PG 94-95; GSS 2.94; Pṛthūdakasvāmin on BSS 12.17; MS 15.50-53; SS 13.25; L 130; GK Śreḍhī 21 (I, p.127); GM 169-170.

X4. Multiplication

Kapātasandhi (a door junction) is of two kinds, and *Gomūtrikā* (A cow's urine) is also of two kinds. In addition, *Tatstha* (being there) is said to be of two kinds, and *Khaṇḍa* (separation) is regarded as of three kinds.

The verse enumerates the names of four methods of multiplication, which are being described in the next four verses. Due to the laconic expressions of the versified rules and the sketchy descriptions of the commentaries, there remains much ambiguity about the details of the procedures. Moreover, the two commentaries (or recensions) do not agree with each other in nomenclature. For example, *Kapāṭasandhi* of Śambhunātha (in A) is called the *koṣṭhabheda* of *Tatstha* by Śambhudāsa (in B), while *Kapāṭasandhi* of the latter is called *Tatstha* by the former. Śambhunātha in the present verse regards *Khaṇḍa* as of two kinds, *rūpavibhāga* and *sthānavibhāga,* but under (A 9) = (X 8) he admits three kinds. On the contrary, stanza (A 9) itself reads *dvividhaḥ* (two kinds) instead of *trividaḥ* (three kinds), although (A 6) = (X 4) reads *tridhā* (three kinds).

Cf. Tr 7.

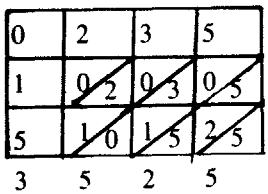
X 5. Kapāṭasandhi ("a door junction")

One should place the price above the question, and multiply with the price in order. There will be two varieties [of the method] called *kapāṭa* according to whether the order [of procedure] is natural *(anuloma)* or inverse *(viloma).*

In the present and the following verses, the multiplier and multiplicand are called "the [quantity in] question" *(praśna)* and "the price" *(mūlya).*

A.

Ex. A horse costs two hundred and thirty-five *korīs*. How much do fifteen horses cost?



235 korīs × 15 = 3525 korīs.

B. Śambhudāsa first explains the four methods for multiplication (X 5-8) and an auxiliary rule concerning zero and unity (X 9), and then gives eleven examples with working processes for only the first example. The first eight examples require the price of a certain thing when its quantity (Q) and the unit price (U) are given. The rest are purely numerical examples which have been devised for a special form *(rūpa)* of the result.

Ex. 1. Q = 1196 gadīyāṇas of *rūpa* (silver?), U = 18 drammas/gadīyāṇa.
Ex. 2. Q = 865 tolas of *hema* (gold), U = 32 ṭaṅkas/tola.
Ex. 3. Q = 196 maṇas of *mañjiṭha,* U = 35 ṭaṅkas/maṇa.
Ex. 4. Q = 4865 maṇas of *dānta* (ivory?), U = 36 ṭaṅkas/maṇa.
Ex. 5. Q = 38327 maṇas of *sāṇḍa,* U = 81 drammas/maṇa.
Ex. 6. Q = 1767 maṇas of *sūkadi,* U = 64 ṭaṅkas/maṇa
Ex. 7. Q = 3703 maṇas of *ācchī,* U = 188 drammas/maṇa.
Ex. 8. Q = 1859 maṇas of *paḍasūtra,* U = 308 ṭankas/maṇa.
Ex. 9. 152207 × 73 = 11111111 (in the form of a *vallī* or creeper).
Ex. 10. 333333666667 × 33 = 11000011000011 (a *kaṇṭhābharaṇa* or necklace).
Ex. 11. 142857143 × 7 = 1000000001 (a *hāra* or necklace of pearls).

Illustration of Ex. 1 (1196 × 18 = 21528). In the ms. the two varieties, the natural and the inverse orders, are illustrated by the "setting" *(nyāsa)*, one of the intermediate steps, and the result (21528), without explanations.
om10t1m10t2m10

	Natural order	Inverse order
Setting	1 1 8	1 8
	1 1 9 6	1 1 9 6
A step	1 8 8 2 8	1 1 9 6 8
	1 9 6	8 7 4
	7 4	8 2
Result	2 1 5 2 8	2 1 5 2 8

These illustrations obviously contain scribal errors. The following are my tentative reconstructions. The underlines indicate the numerals newly written at each step.

(1)	Price	1 8		1 8
	Question	1 1 9 6		1 1 9 6
(2)		1 8		1 8
	18 × 1:	1 8 1 9 6	18 × 6:	1 1 9 6 8
				4
(3)		1 8		1 8
	18 × 1:	1 8 8 9 6	18 × 9:	1 1 9 6 8
		1		7 4
				2
(4)		1 8		1 8
	18 × 9:	1 8 8 2 6	18 × 1:	1 1 9 6 8
		1 9		7 4
		7		8 2
(5)		1 8		1 8
	18 × 6:	1 8 8 2 8	18 × 1:	1 1 9 6 8
		1 9 6		8 7 4
		7 4		8 2
(6)	From (X 9):	1 8		1 8
		1 8 8 2 8		1 1 9 6 8
		1 9 6 0		8 7 4 0
		7 4 0		8 2 0
(7)	Result	2 1 5 2 8		2 1 5 2 8

Steps (1), (5) without the first line, and (7) are given in the ms. At step (6), the empty places are filled with zero according to (X 9).

Cf. Tr 5-6; PG 18-19; GP 10; GSS 2.1; MS 1.3; SS 13.2; GT 15; L 14; GS 1.27; GK Prakīrṇaka 13; GM 17-18.

X 6. Gomūtrikā ("a cow's urine").

One should place the price below the question, and multiply the whole mutually. There will be two varieties [of the method] called *gomūtra* according to whether the order [of procedure] is natural or inverse.

A.

Ex. 1. The area of a square figure.

4 hastas 12 añgulas × 4 hastas 12 añgulas
= 20 sq. hastas 6 sq. añgulas.

4	12
4	
12	4
	12

⇒

4	12
16	
48	48
	144

⇒

4	12
18	
0	54
	0

⇒

4	12	
18	2	20
0	6	6
	0	0

The ms. shows the first step only.

Ex. 2. A horse costs three hundred fifty-five and a half *korīs*. How much are fifteen horses? (28 *dokaḍās* = 1 *korī*)

355 1/2 korīs × 15 = (355 korīs + 14 dokaḍās) × 15
= 5325 korīs + 210 dokaḍās
= 5325 + 7 1/2 korīs = 5332 korīs.

The main text calculates: 355 × 15 + 7 = 5332, while the enclosed expression is:

15 355 ko II[a] dokaḍā 14	 5325[b] 210

a 10 in the ms.
b 5321 in the ms.

B.

Ex. 1196 × 18 = 21528. Just as in the case of *Kapāṭasandhi*, the two varieties are illustrated by the first "setting", one of the intermediate steps, and the result.

	Natural order	Inverse order	
Setting	1 1 9 6	1 1 9 6	(question)
	1 x	x 1	(price)
A step	1 9 7 8 8	1 9 7 8 8	
	1 7 4	1 7 4	
Result	2 1 5 2 5	[2 1 5 2 8]	

Evidently, the result of the natural order is wrong, and that of the inverse order is missing. Likewise, the intermediate step of the inverse order and the second line of the setting in both cases appear to suffer from scribal errors.

(1)	Question	1 1 9 6		1 1 9 6
	Price	1 8		1 8
(2)	1 × 18:	1 8	6 × 18:	6 8
				4
(3)	1 × 18:	1 9 8	9 × 18:	9 8 8
				7 4
(4)	9 × 18:	1 9 7 2	1 × 18:	2 7 8 8
		1 7		7 4
(5)	6 × 18:	1 9 7 8 8	1 × 18:	2 0 7 8 8
		1 7 4		7 4
(6)	From (X 9):	1 9 7 8 8 1 7 4 0		2 0 7 8 8 7 4 0
(7)	Result	2 1 5 2 8	Result	2 1 5 2 8

Cf. BSS 12.55.

X 7. Tatstha ("being there").

A. Śambhunātha's recension reads:

The variety [called] *śīrṣa* (the head) is declared in multiplying the quantity one by one. In addition, the variety [called] *pṛṣṭha* (the back) has been told [in the same category]. [Thus], *tatstha* is also regarded as of two kinds.

Ex. 1. *śīrṣabheda.* 135 × 12 = 1620.

1 2	1 2	1 2	
1 3 5	1 2 3 5	1 5 6 5	1 6 2 0
	(12 × 3)	(12 × 3)	(12 × 5)

Ex. 2. *pṛṣṭhabheda.* 135 × 12 = 1620.

1 3 5	1 3 6 0	1 4 2 0	1 6 2 0
1 2	1 2	1 2	
(12 × 5)	(12 × 3)	(12 × 1)	

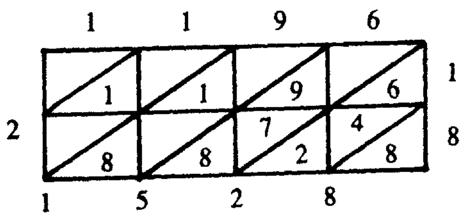
B. Śambhudāsa calls the two varieties "the head" and "the box" (*koṣṭha*).
Ex. 1. *śīrṣabheda*. 1196 × 18 = 21528. In the ms. are given the following three steps.

	Setting	A step	Result
Price	1 8	1 1 9 6 8	2 1 5 2 8
Question	1 1 9 6	8 8 7	

My tentative reconstruction is as follows:

(1)	1 × 1196:	1 1 9 6
(2)	8 × 1:	1 1 9 6 8
(3)	8 × 1:	1 1 9 6 8 8
(4)	8 × 9:	1 1 9 6 8 8 2 7
(5)	8 × 6:	1 1 9 6 8 8 8 2 7 4
(6)	From (X 9):	1 1 9 6 8 8 8 2 0 7 4 0
(7)	Result:	2 1 5 2 8

Probably the intermediate step in the ms. is a corrupted form of step (5).
Ex. 2. *koṣṭhabheda*. 1196 × 18 = 21528.



Cf. Tr 6; PG 19; GSS 2.1; GM 16.

X 8. Khaṇḍa ("part")

In some cases the separation by integers (*rūpavibhāga*) will be [employed], in other cases the separation by places (*sthānabhāga*), and still in other cases the separation by decreasing and increasing (*hīnādhikabhāga*). The [method called]

khaṇḍa has also been handed down [to us] in three kinds.
(1) Separation by integers: $ab = ax + ay$, where $b = x + y$.
A.
Ex. Given under (A 6) = (X 4).

$$135 \times 12 = 135 \times 6 + 135 \times 6 = 810 + 810 = 1620.$$

Expression in the ms.:

6	135	810	sum	multiplier	12	product
6	135	810	1620	multiplicand	135	1620

B.
Ex.

$$1196 \times 18 = 1196 \times 9 + 1196 \times 9 = 10764 + 10764 = 21528.$$

2) Separation by places:

$$ab = ax_1\, 10^n + ax_2 10^{n-1} + \ldots. + ax_{n-1}\, 10^1 + ax_n 10^0,$$

where $b = x_1\, 10^n + x_2 10^{n-1} + \ldots + x_{n-1} 10^1 + x_n 10^0$.
A.
Ex. $135 \times 12 = 135 \times 10 + 135 \times 2 = 1350 + 270 = 1620.$
B.
Ex. $1196 \times 18 = 1000 \times 18 + 100 \times 18 + 90 \times 18 + 6 \times 18$
$= 18000 + 1800 + 1620 + 108 = 21528.$

Expression in the ms.:

18				
1000		18000		
18				
100	⇒	1800	⇒	21528
18				
90		1620		
18				
6		108		

3) Separation by decreasing and increasing:
A. According to Śambhunātha, the rule given is:

$$ab = a\,(b \pm k) \mp ak.$$

Ex. 1. $997 \times 25 = (1000 - 3) \times 25 = 25000 - 75 = 24925.$
Ex. 2. $293 \times 50 = (290 + 3) \times 50 = 14500 + 150 = 14650.$

B. According to Śambhudāsa, the rule given is:

$$ab = (ak)\,(b/k).$$

Ex. $18 \times 1196 = (18 \times 2) \times (1196/2) = 36 \times 598 = 21528.$

Cf. BSS 12.55-56; Tr 7; PG 20; GSS 2.1; GT, p.5; L 14-15; GS 1.29; GK Prakīrṇaka 14-15; GM 16.

X 9. Multiplication by zero and unity

[A number], multiplied by zero, is zero. One should employ zero in front. [A number], multiplied by unity, is the same. This will be so indeed everywhere. Three rules are prescribed here.

1) $a.0 = 0$.
2) Write down zero in front.
3) $a. 1 = a$.

A. Śambhunātha differentiates a.0 = 0 and 0.a = 0 in the first rule, and says that "it is the same" in the case of unity (the third rule) as well, but does not refer to the second rule. His reading of the second quarter of the verse itself is quite ambiguous: "One should employ (what?) in the way (?) of zero" (*śūnyamārge niyojayet*).

B. According to Śambhudāsa, the second rule is meant for filling empty places in multiplication. See Exs. for (X5) and (X 6) above.

For zero, cf. BSS 18.32-35; Tr 8 = PG 21; GP 17; GSS 1.49; MS 15.10-11; SS 14.6; GT 45; L 45-46; BG 16-20; BGA 1.11-15; GS 1.31; GK Prakīṇaka 30; GM 46.

X 10. Division

Having put down the divisor below the question (i.e., the dividend), and divided the question by the divisor, the division should be made (lit. part should be taken away) in order. [Thus] the rule for division has been certainly handed down [to us].

This gives the well-known method, which places the divisor (*hara*) under the dividend, which (the latter) is called "the [number in] question" (*praśna*) in our text.

A.

Ex. 1620 ÷ 12 = 135.

$$\begin{array}{ccccccc} & & \underline{1} & & 1\,\underline{3} & & 1\,3\,\underline{5} \\ \underline{1\,6\,2\,0} & \Rightarrow & \underline{4}\,2\,0 & \Rightarrow & \underline{6}\,0 & \Rightarrow & \underline{0} \\ \underline{1\,2} & & \underline{1\,2} & & \underline{1\,2} & & 1\,2 \end{array}$$

The last step and the first line of the second and the third steps are not given in the ms.

B. After having given an auxiliary rule (B 9U), Śambhudāsa assigns ten examples, which all require division of a certain amount of money (the denomination is always the *dramma*).

Ex. 1. 488/4 = 122.
Ex. 2. 327/3 = 109.
Ex. 3. 4096/16 = 256.
Ex. 4. 11664/108 = 108.
Ex. 5. 30276/87 = 348.

Ex. 6. 156025/395 = 395.
Ex. 7. 1466521/1211 = 1211.
Ex. 8. 19345600/1808 = 10700.
Ex. 9. 10000003/13 = 769231.
Ex. 10. 100001/11 = 9091.

Śambhudāsa only gives the first "setting" (*nyāsa*) and the quotient. Thus,

4	8	8
4		

and 122, for the first example.

Cf. Tr 9; PG 22; GP 11; GSS 2.18-19; MS 15.4; SS 13.3; GT, p.6; L 18; GS 1.33; GK Prakīrṇaka 16; GM 19.

B 9U. Zero for a quotient

[When] there is no division, the quotient is zero.
This rule is no doubt meant for filling empty places in the line of the quotients obtained at each step, but it has been rarely stipulated.

X 11. Square and cube

In the case of the product of two equal quantities, there is a square only. In the case of the product of three equal terms, there will be certainly a cube.

$a^2 = a.a. \quad a^3 = a.a.a.$

A.
Exs. $2^2 = 4,\ 3^2 = 9,\ \ldots,\ 9^2 = 81,\ 1^2 = 1.$
$3^3 = (3.3).3 = 9.3 = 27,\ 1^3 = 1,\ 2^3 = 8,\ 4^3 = 64,\ \ldots,\ 9^3 = 729.$
B.
Exs. $1^2 = 1,\ 2^2 = 4,\ \ldots,\ 5^2 = 25,\ 15^2 = 225,\ 25^2 = 625,\ 235^2 = 55225.$
$1^3 = 1,\ 2^3 = 8,\ \ldots,\ 5^3 = 125,\ 15^3 = 3375,\ 25^3 = 15625.$

Cf. Abh 2.3; BSS 12.62; Tr 11,15; PG 24,28; GP 11; GSS 2.29,43; MS 15.6; SS 13.4; GT 21,26; L 19,24; GS 1.35,41; GK Prakīrṇaka 17,21; GM 23,27.

X 12-13. Square-root

Having subtracted a square from the [highest] odd [place], one should divide the next [place] by twice [the square-root], put the quotient into the line [of the results], and subtract its square from the next. Having doubled [the square-root] as before, and slid that [divisor to the next place], one should divide the next. This [procedure] should be done at the succeeding [places] also. One should have what has been doubled.

This gives the well-known algorithm. No example in A.

B.

Ex. $\sqrt{1} = 1, \sqrt{4} = 2, \sqrt{9} = 3, \sqrt{16} = 4, \sqrt{25} = 5, \sqrt{225} = 15, \sqrt{625} = 25, \sqrt{55225} = 235$ (without working process).

The working process for the last example may be reconstructed as follows.

o e o e o	(o = odd, e = even)	5 5 2 2 5
$\underline{5\,5\,2\,2\,5}$		
$\underline{1}\,5\,2\,2\,5$	$(5 - \underline{2}^2 = 1)$	
$\underline{2}$		
1 5 2 2 5		
$\underline{4}$	$(2.2 = \underline{4})$	
$\underline{3}\,2\,2\,5$	$(15 - 4.\underline{3} = \underline{3})$	
$4\,\underline{3}$		
$\underline{2}\,\underline{3}\,2\,5$	$(32 - 3^2 = 2\underline{3})$	
4 3		
2 3 2 5		
$\underline{4\,6}$	$(3.2 = \underline{6})$	
2 3 2 5		
$\underline{4}\,\underline{6}$	(slide)	
$\underline{2}\,5$	$(232 - 46.\underline{5} = \underline{2})$	
$4\,6\,\underline{5}$		
$\underline{0}$	$(25 - 5^2 = \underline{0})$	
4 6 5		
0		
$4\,\underline{7}\,\underline{0}$	$(5.2 = \underline{10})$	
0		
$\underline{2}\,\underline{3}\,\underline{5}$	$(470/2 = \underline{235})$	

Cf. Abh 2.4; Tr 12-13; PG 25-26; GP 12; GSS 2.36; MS 15.6-7; SS 13.5; GT 23; L 22; GS 1.37-38; GK Prakīrṇaka 19-20; GM 25.

X 14. Multiplication and division of fractions

A. Śambhunātha's recension is as follows.

A 15. The product of the numerators, divided by the product of the denominators, is the result in the case of multiplications of fractions. In the case of division, having turned the numerator and the denominator [of the divisor] upside down, [one should obtain] the result just as in the case of multiplications.

$$\frac{b}{a} \cdot \frac{d}{c} = \frac{bd}{ac} .$$

$$\frac{b}{a} \div \frac{d}{c} = \frac{b}{a} \cdot \frac{c}{d} = \frac{bc}{ad} .$$

Ex. 1. $\frac{10}{4} \cdot \frac{10}{4} = \frac{100}{16} = 6\frac{1}{4}$.

Ex. 2. $6\frac{1}{4} \div \frac{10}{4} = \frac{25}{4} \cdot \frac{4}{10} = \frac{100}{40} = 2\frac{1}{2}$.

Ex. 2 is identical with Tr Ex. 10ab.

Cf. BSS 12.3-4, 60; BM Q3, Q4; Tr 20; PG 33; GP 16; GSS 3.2,8; MS 15.15; SS 13.9-10; GT 35-36 (pp.19-21); L 39, 41; GS 1.53,55; GK Prakīrṇaka 28-29; GM 43.

B. The rule in Śambhudāsa's recension is considerably different. It is meant for the multiplication of those numbers which consist of an integer part of a certain measure and its fractional part.

B 13. The integer should be correctly multiplied by the [other] integer, and both fractional parts [produced] from division also correctly by it. Those fractional parts are [multiplied] mutually. One should obtain the result produced from fractions [in this way].
This rule may be expressed thus:

$$(a;b).\ (c;d) = ac + (0;ad) + (0;bc) + (0;0,\ bd),$$

where a and c are integer portions, and b and d are fractional portions, of a certain measure.
Ex. 1. Area of a rectangular field. 10 gajas 12 aṅgulas by 1 gaja 8 aṅgulas, where 24 *jyeṣṭha* (largest) aṅgulas = 1 gaja.

10;12.1;8 = 10.1 + 10.0;8 + 1.0; 12 + 0;12.0;8
= 10 + 3;8 + 0;12 + 0;4 = 14 sq. gajas.

Ex.2. Area of a rectangular field. 25 gajas 14 visās by 16 gajas 10 visās, where 20 visās = 1 gaja.

25;14.16;10 = 400 + 0;224 + 0;250 + 0;0, 140
= 400 + 11;4 + 12;10 + 0;7 = 424;1 sq. gajas.

X 15. Rule of three (trairāśika) and inverse rule of three

A. Śambhunātha's recension can be translated thus:

A 16. The product of the middle and the last [terms] is first [made, and then] divided by the first in the rule of three. In the inverse [rule of three], the product of the middle and the first, divided by the last, will be the result [to be obtained].

When $a_1: b = a_2: x$,

$x = ba_2/a_1$ (Rule of three).

When $a_1 : a_2 = x:b$

$x = a_1 b/a_2$ (Inverse rule of three).

In both cases, the three terms are arranged thus: (a_1, b, a_2), where a_1 and a_2 must be measured by the same unit.

Rule of three.

Ex. 1. One *sera* of ghee (*ghṛta*) is obtained for eight *dokaḍās*. How much is obtained for twenty *dokaḍās?*

(8,1,20) = 20/8 = 2 1/2 *seras*.

Inverse rule of three.

According to Śambhunātha, the inverse rule of three can be applied to the following relationships.

Age: price of living creatures.
Purity: weight of gold.
Shadow: time from a gnomon.
Size: times for filling/emptying measuring cups.

Ex.2. A *baladī* of six *dharas* costs twenty *korīs*. How much does that of ten *dharas* cost?

(6,20,10) = 120/10 = 12 korīs.

Ex.3. A female slave (*dāsī*) who is twenty years old costs a hundred *korīs*. How much is one twelve years old?

(20,100,12) = 2000/12 = 166 2/3 korīs.

According to Gaṇeśa of Nandigrāma (on L 79), a girl sixteen years old is the most expensive, which contradicts the present example. Cf. Tr Ex. 56; PG Ex. 50; GT 109; al-Bīrūnī's India, I, p. 313.

Ex.4. One makes eleven *tolas* of pure gold (where complete purity is sixteen sixteenths) into a piece of gold whose purity is thirteen. What is the weight of the mixture?

(16,11,13) = 176/13 = 13 7/13 tolas.

Cf. Abh 2.26-27; BSS 12.10-11; BM C10, N19,Q11; Tr 29-30; PG 43-44; GP 24; GSS 5,2; MS 15.24-25; SS 13.14; GT 86; L 73; GS 1.63,81; GK Prakīrṇaka 60-61; GM 88.

B. In Śambhudāsa's recension, the three terms, a_1, b, and a_2, are called, in this order, "the last", "th middle", and "the first". This order of naming is the reverse of the usual one, but the right-to-left order itself is popular in the decimal place-value system. Thus, for example, the "2" and the "4" of the number 234 are designated "the last" and "the first" digits respectively.

B 14. The product of the middle and the first is first [made, and then] divided by the last in the rule of three. In the inverse case, the product of the middle and the last, divided by the first, will be the result [to be obtained].

Rule of three.

Ex. 1. Five *maṇas* of rice are obtained for twenty-one *drammas*. How much do ninety *maṇas* of the same cost?

(5,21,90) = 378 drammas.

Ex. 2. Six *maṇas* of *mudga* beans are obtained for twenty-three *drammas*. How much of the same thing is obtained for eighty-one *drammas*?

(23,6,81) = 21 3/23 maṇas = 21 maṇas 5 5/23 seras.

Inverse rule of three.

Ex. 3. A woman *(kamārī)* eighty years old costs forty *tankas*. How much does a woman sixteen years old cost?

(80,40,16) = 200 ṭaṅkas.

Ex. 4. A certain pearl is obtained for twenty-five *drammas* of one hundred *caḍatu ṭaṅkas*. For how many *drammas* of sixty *caḍatu ṭaṅkas* is the same pearl obtained?

(100,25,60) = 41 2/3 drammas = 41 drammas 2 jathalas.

X 16. Investments (prakṣepa)

A. Śambhunātha's recension is:

A 17. By the profit produced from various investments, they (i.e., the investments) are multiplied [severally], and divided by their sum. [The result] will be the shares of each [investor].

When a_1 is the investment of the *i*-th investor ($i = 1,2,\ldots, n$), M, the profit obtained from the investments, then the share of the *i*-th investor is:

$$pi = \frac{a_i . M}{A}, \text{ where } A = a_1 + a_2 + \ldots. + a_n.$$

Ex. Three persons invested severally fifty-one, sixty-eight, and eighty-five *korīs*, and obtained three hundred *korīs* in total. How much is the share of each?

Investments *(a_i)*: 51, 68, 85 korīs.

Profit *(M)*: 300 korīs.

$A = 51 + 68 + 85 = 204.$

$$\frac{51.300}{204} = \frac{15300}{204} = 75 \text{ korīs.}$$

$$\frac{68.300}{204} = \frac{20400}{204} = 100 \text{ korīs.}$$

$$\frac{85.300}{204} = \frac{25500}{204} = 125 \text{ korīs.}$$

B. Śambhudāsa's recension is:

B 15. The product of various investments and the profit is [made] severally. From its division by the sum [of the investments will be] two quotients. The result will be that [share] of each [investor].

According to Śambhudāsa, the rule prescribes two kinds of procedures.

$$p_i = \frac{a_i M}{A}, \quad \text{and} \quad p_i = \frac{M}{A} a_i.$$

But it is difficult (if not impossible) to see the latter formula in his wording of the verse.

Ex. 1. Seeds.
Investments (a_i): 2, 3, 4, 5 seīs.
Profit (M): 210 seīs.
Method 1. $A = 2 + 3 + 4 + 5 = 14$ seīs.
a_iM: 420, 630, 840, 1050.
p_i: 30, 45, 60, 75 seīs.
Method 2. $A = 2 + 3 + 4 + 5 = 14$ seīs.
$M/A = 210/14 = 15$.
p_i: 30, 45, 60, 75 seīs.

This example is identical with *Triśatikā* Ex. 67, although the latter uses *prastha* instead of *seī*.

Ex. 2. Money.

Investments (a_i): 100, 500, 1000 ṭankas.
Profit (M): 200 ṭankas.

Method 1. $A = 100 + 500 + 1000 = 16000$ ṭankas.
$a_i M$: 20000, 100000, 200000.
p_i: 12 1/2, 62 1/2, 125 ṭankas.

Method 2. $A = 100 + 500 + 1000 = 16000$ ṭankas.
$M/A = 200/1600$ ṭankas/ṭankas
$= 200.48/1600$ jayasthalas/ṭankas.
$= 6$ jayasthalas/ṭankas.
p_i: 600, 3000, 6000 jayasthalas, or
12 1/2, 62 1/2, 125 ṭankas.
(48 jayasthalas = 1 ṭanka.)

This example is identical with *Triśatikā* Ex. 68-69, although the latter does not specify the denomination.

Cf. BSS 12.16; BM N1; Tr 38; PG 59; GSS 6.79; MS 15.36; SS 13.19; L 94; GS 3.7; GK Miśraka 1; GM 111.

X 17. Mixture of gold (miśraka)

A. Śambhunātha's recension reads:

A 18. One should divide the sum of the products of the colours (i.e., the purities)

and [the weights of] the gold [pieces] by the sum of [the weights of] the gold [pieces]. The colour [of the mixture] should be known in the numerical figure obtained. When [the first sum is] divided by [the weight of] the gold [obtained], the colour [of the purified gold is obtained].

When v_i and w_i are, respectively, the purity (*varṇa*) and the weight (*suvarṇa*) of the *i*-th piece of gold ($i = 1,2,...n$), then the purity of the mixture of them is:

$$V = \frac{v_1w_1 + v_2w_2 + \ldots + v_nw_n}{W}, \text{ where } W < w_1 + w_2 + \ldots + w_n.$$

$$V = \frac{v_1w_1 + v_2w_2 + \ldots + v_nw_n}{W}, \text{ where } W < w_1 + w_2 + \ldots + w_n.$$

Śambhunātha's example (see below) suggests that the second formula is meant for the case where the weight (W) of the mixture decreases due to purification (otherwise the rule is superfluous), but this condition is not stated in the verse.

Ex. Purity (v_i): 13, 12, 11, 10 varṇas.

Weight (w_i): 10, 4, 2, 4 māsas.

$$V = \frac{13.10 + 12.4 + 11.2 + 10.4}{10+4+2+4} = \frac{240}{20} = 12 \text{ varṇas.}$$

When the mixture has lost its weight by purification, and becomes 16 *māṣas*.

$$V = \frac{240}{16} = 15 \text{ varṇas.}$$

B. Śambhudāsa's recension lacks words for the "products" (v_iw_i) and the "sum" (W), which are indispensable for the rule.

B 16. The sum of [the products of] various colours and [the weights of] the gold [pieces] is divided by [the sum of the weights of] the gold [pieces]. The colour [of the mixture] should be known in the numerical figure obtained. When [the first sum is] divided by the colour [of the mixture], the weight [of the mixture is obtained.]

Here the second rule is:

$$W = \frac{v_1w_1 + v_2w_2 + \ldots + v_nw_n}{V}.$$

This rule is useful only for the case of purification (otherwise the weight of the mixture is simply the sum of the weights of the gold pieces given), but Śambhudāsa applies it to the case of a simple mixture in his example.

Ex. Purity (v_i): 12, 14 varṇas.

Weight (W_i): 8, 4 gadīyāṇas.

$$V = \frac{12.8 + 14.4}{8 + 4} = \frac{152}{12} = 12\frac{2}{3} \text{ varṇnas.}$$

$$W = \frac{152}{12\ 2/3} \simeq \frac{152}{13} = 11\frac{9}{13} \text{ gadīyāṇas.}$$

The decrease of weight, 4/13 *gadīyāna,* is due, of course, to the approximation of the denominator, 12 2/3, by 13, and not to the purification of the gold.

Cf. BM 27-29; Tr 36-37; PG 52-53; GSS 6.169, 172; MS 15.38-39; L 103; GS 3.15, 18; GK Miśraka 18; GM 138.

X 18. Areas of a square, an oblong, and a trapezoid.

In the case of a quadrilateral, equal-[sided] (square) or oblong, the product of the side and the upright is the area. Half of the sum of the base and the face, multiplied by the perpendicular, is the area in the case of an odd [quadrilateral] (i.e., a trapezoid).

$$A = ab,$$

where *a and b* are, respectively, the side and the upright of a square *(sama-caturasra)* or an ablong *(dīrgha-caturasra)*.

$$A = \frac{a+b}{2}. h,$$

where $a, b,$ and h are the mouth or face *(mukha)*, the earth *(vasudhā)*, and the perpendicular *(lamba)* of a trapezoid *(viṣama-caturasra,* lit. an irregular quadrilateral).

A.

Ex. 1. A Square. $a = b = 4$. $A = 16$.

Ex. 2. An oblong. $a = 8$, $b = 3$. $A = 24$.

Ex. 3. A trapezoid. $a = 8$, $b = 20$, ($c = 5$, $d = 6$,) $h = 4$. $A = 56$.

The unit of measurement is not explicitly given in most of Śambhunātha's geometrical examples, but it is probably the *aṅgula* unless otherwise indicated. See under (X 20). Ex. 1 is identical with Tr Ex. 75ab.

B.

Ex. 1. A square. $a = b = 4$ gajas. $A = 16$ sq. gajas.

Ex. 2. A square. $a = b = 4\ 1/2$ gajas. $A = 20\ 1/4$ sq. gajas.

Ex. 3. An oblong. $a = 4$, $b = 2$ gajas. $A = 8$ sq. gajas.

Ex. 4. An oblong. $a = 4\ 1/2$, $b = 2\ 1/2$ gajas. $A = 11\ 1/4$ sq. gajas.

Ex. 5. A trapezoid. $a = 4$, $b = 14$, ($c = d = 13$,) $h = 12$ gajas.
$A = 108$ sq. gajas.

Ex. 1 is identical with Tr Ex. 75ab, and Ex. 5 with Tr Ex. 78.

Cf. Abh 2.8; Tr 42; PG 115; GSS 7.50; MS 15.78; SS 13.30; L 173; GS 3.35,39; GK Kṣetra 34,45; GM 231, 240.

X 19. Areas of a triangle and a circle

Half of the base, multiplied by the perpendicular, is the area of a triangular field. The product of a quarter of the diameter and of the circumference is [the area] of a circle.

$$A = \frac{a}{2}.h \text{ (X 19ab)},$$

where a and h are, respectively, the earth (*vasudhā*) or the base and the perpendicular (*lamba*) of a triangle (*trikoṇaka*).

$$A = \frac{d}{4}.c \text{ (X 19cd)},$$

where c and d are, respectively, the circumference (*vṛtti*) and the diameter (*vyāsa*) of a circle (*vartula*).

The reconstructed ancestor (X 20 and 23) and Śambhudāsa (B 18U), and most probably the original *Pañcaviṃśatikā* as well, employ 19/6 for π, while Śambhunātha uses $\sqrt{10}$ (A 22 and 24) in addition to 19/6 (A 21 and 23). Ṭhakkura Pherū also uses both $\sqrt{10}$ and 19/6 side by side (GS 3.43 and 45). It is likely that the fraction, 19/6, was obtained from $\sqrt{10}$ by means of the well-known approximation formula: $\sqrt{a^2 + r} \simeq a + r/2a$.

A.

Ex. 1. An isosceles triangle. $a = 6$, ($b = c = 5$,) $h = 4$. $A = 12$.

Ex. 2. A circle. $d = 7$.

c. [$= (19/6).7 = 22\ 1/6$] $\simeq 22$, $A = (7/4).22 = 38\ 1/2$.

Here Śambhunātha does not give the computation of the circumference (c), which has been reconstructed according to (A 21). It is difficult to tell why Śambhunātha did not choose 6, instead of 7, for the diameter (d), just as Śambhudāsa did (see below). He uses the same diameter in (A 21, 22, and 23). The diameter and the circumference, 7 and 22, suggest that he simply borrowed the present example from a book which employed the famous approximation to π, 22/7; this value was known to Bhāskara II and Āryabhaṭa II, and perhaps to Śrīdhara as well.

B.

Ex. 1. An isosceles triangle. $a = 14$, ($b = c = 13$), $h = 12$ gajas.
$A = 60$ sq. gajas.

Ex. 2 A circle. $d = 6$ gajas.

$c = 19$ gajas (according to (B 18U), $A = 28\ 1/2$ sq. gajas.

For (X 19ab), cf. L 166; GS 3.39; GK Kṣetra 39; GM 219. For (X 19cd), cf. Abh 2.7; GSS 7.60; L 201; GS 3.43,45; GK Kṣetra 9.

A 21. Circumference and area of a circle

The diameter, multiplied by three and increased by one sixth of the diameter, is the circumference [of a circle]. One should multiply mutually half of the diameter and half of the circumference. [The result is the area of the circle.]

$$c = 3d + \frac{d}{6} \; [= \frac{19}{6} d] \quad \text{(A 21ab).}$$

$$A = \frac{d}{2} \cdot \frac{c}{2} \text{(A 21cd).}$$

Ex. $d = 7$. $c = 3.7 + 7/6 = 22\ 1/6 \simeq 22$.
$A = (7/2).(22/2) = 38\ 1/2$.
(A 21) = AP 47.22 = GS 3.45 in contents.

AP 47.22:

vyāse triguṇe paridhiḥ ṣaṣṭhabhāgavimiśite/
vyāsārdhaṃ ca paridhyardhaṃ guṇe vṛttaphalaṃ bhavet//

GS 3.45:

vaṭṭassa ya vikkhaṃbhaṃ tiuṇaṃ taha chaṭṭhamaṃsajuya parihī/
vikkhaṃbhaddhe guṇiyā parihi dalaṃ tassa khittiphalaṃ//

Cf. GS Uddeśapañcaka 22:

vaṭṭassa ya vikkhaṃbhaṃ tiuṇaṃ taha chaṭṭhamaṃsajuya parihī/
sā pāya vitthare guṇi jaṃ jāyaī taṃ ji khittaphalaṃ//

Cf. also (B 18U) below. For (A 21cd), see (X 19) above.

A 22. Area of a circle

When the square of the diameter is decreased by one fifth [of itself], [the result is] roughly the area. The square of the square of half the diameter is multiplied by ten. From its square-root the area [of the circle is obtained].

$$A = d^2 - \frac{d^2}{5} \; [= \frac{16}{5} \cdot \frac{d^2}{4}] \quad \text{(A 22ab).}$$

$$A = \sqrt{10\,(d/2)^4} \; [= \sqrt{10} \cdot \frac{d^2}{4}] \quad \text{(A 22cd).}$$

Ex. A circle. $d = 7$.
Method 1. $d^2 = 49$, $d^2/5 = 9\ 4/5 \simeq 9\ 3/4$,
$A = 49 - (9\ 3/4) = 39\ 1/4$.

Method 2. $A^2 = 10\ (7/2)\ 4 = 24010/16 = 1500\ 5/8 \simeq 1500.$

A rule (A 22U) is quoted here in order to compute the square root of 1500.

$1500 = 38^2 + 56,$

$$\frac{60\ (56 + 1)}{2.38 + 2} = \frac{3420}{78} = 43\frac{11}{13} \simeq 44.$$
$A \simeq \sqrt{1500} \simeq 38 + 44/60 \simeq 38\ 3/4.$

For (A 22cd), cf. BSS 12.40; GSS 6.60; Tr 45; MS 15.88; SS 13.35.

A 22U. Approximation to the square-root

The remainder of the square-root is increased by unity, multiplied by sixty, increased by the fractional portion (*vikalās*), and divided by twice the square root plus two. The quotient will be accurate [approximation to the fractional portion of the square root].

This rule is meant for the case where the fractional portion of the number in question (n) is expressed in the sexagesimal system, say,

$$n = k + \frac{v}{60}\ (k \text{ and } v \text{ are positive integers, and } 0 \leqq v < 60).$$

Let a be the nearest square root in integers,

$$k = a^2 + r, \text{ or } n = a^2 + r + \frac{v}{60},$$

then the rule may be expressed thus:

$$\text{The fractional portion of } \sqrt{n} \simeq \frac{60\ (r + 1) + v}{2a + 2}.$$

Or,

$$\sqrt{n} \simeq a + \frac{60\ (r+1) + v}{2a + 2} \cdot \frac{1}{60}\ [\ = a + \frac{r + v/60 + 1}{2\ (a + 1)}\ ].$$

Cf. BSS 12.64-65; BM Q2. Cf. also Tr 46, PG 118, MS 15.55, SS 13.36, L 140, and GK Kṣetra 30-31, which all prescribe approximation formulas of the type:

$$\sqrt{n} = \frac{\sqrt{np^2}}{p},$$

where $\sqrt{np^2}$ is obtained by means of the algorithm based on the place-value system. Cf. (X 12-13) above.

A 23. Area of a circle

The square of the diameter, when decreased by one fifth [of itself], is known to be the area. Or, otherwise, to the square quantity of the diameter, decreased by its quarter, one should add one eighteenth of itself. [The result] is known to be the area.

$$A = d^2 - \frac{d^2}{5} \quad [= \frac{16}{5} \cdot \frac{d^2}{4}] \qquad \text{(A 23ab).}$$

$$A = d^2 - \frac{d^2}{4} + (d^2 - \frac{d^2}{4}) \cdot \frac{1}{18} [= \frac{19}{6} \cdot \frac{d^2}{4}] \qquad \text{(A 23 cdef).}$$

Ex. A circle. $d = 7$.
Method 1. See under (A 22) above.
Method 2. $d^2 - d^2/4 = 49 - (12\ 1/4) = 36\ 3/4$,
$A = 36\ 3/4 + (36\ 3/4)/18 = 36\ 3/4 + 4 + 2\ 1/24 \simeq 38\ 3/4$.

Verse (A 23ab) is identical with (A 22ab) above. (A 23cdef) is identical with AP 47.24 in contents, and AP 47.24 gives another formula equivalent to it.

AP 47.24:

vyāse vargasya guṇite[a] caturthāṃśaṃ tu pātayet/
śeṣe ' ṣṭādaśake bhāge kṣipte vṛttaphalaṃ bhavet//
[a]Read vyāse vargīkṛte tasya.

AP 47.25:

vyāsavargahastaikonapañcāngulāni[a] pātayet/
śeṣaṃ yal labhyate tac ca vṛttasyāpi phalaṃ bhavet//
[a]Read — hastaikaśaḥ pañcāṅgulāni.

$$A = d^2 - \frac{5}{24} d^2 [= \frac{19}{6} \cdot \frac{d^2}{4}].$$

A 24. Area of a segment of a circle.

Half of the sum of the cord and the arrow is multiplied by the arrow. The product of its square and ten is divided by nine. Its square-root is regarded as the area of the bow and arrow (i.e., the segment of a circle).

$$A = \sqrt{10[h(a+h)/2]^2/9} [= \sqrt{10} \cdot \frac{h\,(a+h)}{6}].$$

where a and h are, respectively, the cord (*jyā*) and the arrow (*śara/bāṇa*) of a segment of a circle.

Ex. A segment of a circle. $a = 6$, $h = 1$
$A^2 = 10[1.(6+1)/2]/9 = 490/36 = 13\ 11/18 \simeq 13$.
$A = \sqrt{13}$.

Śambhunātha expresses the square-root of 13 as "315", where the vertical stroke (I) means a quarter of the unit of length, which is not specified here as elsewhere, and the "5" is the number of its sub-unit. If the conversion ratio is k, then "315" means "3 units and $(k/4+5)$ sub-units".

If we apply the above rule for root approximation (A 22U), we obtain, $\sqrt{13} \simeq 3 + 5/8$. From the equation,

$$\frac{k}{4} + 5 = \frac{5}{8}k,$$

we obtain: $k = 40/3 = 13\ 1/3$. This is, however, impossible not only because k must be an integer but also because the expression "315" in the manuscript implies that $k/4 > 5$, or $k > 20$. Hence we have to admit either that a different method was used for the root-approximation, or that the expression "315" has been corrupted, or both.

For the same formula, cf. Tr 47; MS 15.89; GS 3.46, See also (X 20) below.

B 18U. Circumference of a circle

Three times the diameter, increased by one sixth [of a diameter], is the circumference [of a circle].

$$c = 3d + \frac{d}{6} [= \frac{19}{6} d].$$

This formula is employed in Śambhudāsa's Ex.2 for (X 19) =(B 18).

Cf. (A 21) above.

B 19. Heron's formula for a trilateral and a quadrilateral.

Half of the sum of [all] the sides is [put down] four times, and [severally] decreased by the sides. The square-root of their product will surely be the area of a trilateral or of a four-faced [figure] (i.e., a quadrilateral).

$$A = \sqrt{(s-a)(s-b)(s-c)(s-d)},$$

where a, b, c, and d are the sides of a quadrilateral ($d = 0$ in the case of a trilateral), and $s = a + b + c + d$.

Ex.1 (=Ex.5 for (X 18)). $s = 22$, $A = 108$ sq. gajas.

Ex.2 (=Ex.1 for (X 19)). $s = 18$, $A = 60$ sq. gajas.

Cf. BSS 12.21; Tr 43; PG 117; GSS 7.50; MS 15.69; SS 13.28; L 169; GK Kṣetra 33-34; GM 224.

X 20. Area of a segment of a circle, and that of "a rim" and of any other irregular figure

The product of the arrow and half the sum of the cord and the arrow is increased by one eighteenth [of itself]. In the case of a rim-like figure and an irregular [figure], the product of the breadth and the length is the area.

$A = h(a+h)/2 + h(a+h)/2.1/18$ [$= 19/6.\ h(a+h)/6$] (X 20 ab).
where a and h are, respectively, the cord (*jyā*) and the arrow (*śara/iṣu*) of a segment of a circle.

$A = ab$ (X 20cd),

where a and b are, respectively, the breadth (*vyāsa*) and the length (*dīrgha*) of 'a rim' (*nemyākṛti*), i.e., an annulus, or of any irregular (*viṣama*) figure.

A. Śambhunātha refers to the relationship,

A sq. angulas $= A/576$ sq. hastas,

where 24 angulas = 1 hasta. He gives no examples. Cf. (A 24) above for a segment of a circle.

B.

Ex.1. A segment of a circle. $a = 14$, $h = 4$ gajas. $A = 38$ sq. gajas.

Ex.2. A rim. $a = 3$, $b = 5$ gajas. $A = 15$ sq. gajas. This is identical with the later half of *Triśatikā* Ex. 82.

Śambhudāsa gives here five examples of irregular figures, which are all mentioned in the *Triśatikā* (under 44 and 48), and some of them are identical in both works. Irregular figures are dealt with also in GSS 7.7-38 and GK 4.13-14.

Ex.3. A drum-like figure (*murajakṣetra*), which consists of one oblong and two segments of a circle. $a = 10$, $b = 2$, $h = 3$ gajas. $A =$ 61 1/6 sq. gajas = 61;4 sq. gajas.

Ex.4. A barley-corn (*yava*)-like figure, which consists of two segments of a circle. $a = 10$, $h = 4$ gajas. $A = 59$ 1/9 sq. gajas = 59;2,16 sq. gajas.

Ex.3. A diamond (*vijra*)-like figure, which consists of two trapezoids.
$a = 3$, $b = 5$, $h = 4$ gajas. $A = 32$ sq. gajas.

This is identical with *Triśatikā* Ex. 84

Ex.4. A half-moon (*ardhacandra*)-like figure, which consists of two triangles. $a = 3$, $h = 8$ gajas. $A = 24$ sq. gajas.
This is identical with *Triśatikā* Ex. 83, where the figure is called "the young moon" (*bālendu*).

Ex.5. A pentagon (*pañcabhujakṣetra*), which consists of an oblong and a triangle upon it. $a = 4$, $b = 2$, $h = 3$ gajas. $A =$ 14 sq.gajas.

For formulas similar to (X 20 ab), cf. GP 25; MS 15.93-94; Gaṇeśa on L 213. See also (A 24) above. For (X 20cd), cf. Abh 2.9.

X 21. Volume of a rectangular solid

Of a ditch, a timber, a stone, a storehouse, or a pile [of bricks], the volume is the product of the breadth and the length, multiplied by the thickness, if it is even (i.e., a smooth rectangular solid).

$V = abc$,

where a, b and c are the breadth (*vyāsa*), the length (*dīrgha*), and the thickness (*piṇḍa*) of a rectangular solid such as a ditch, a timber, a stone, or a storehouse.

A. Śambhunātha gives no examples.

B.

Ex.1. $a=4$, $b=5$, $c=3$ gajas, $V=60$ cub. gajas.
Ex.2. $a=4$ gajas 5 visās, $b=5$ gajas 10 visās, $c=3$ gajas 8 visās.
$V=$ 79; 9,10 cub. gajas.
Cf. BSS 12.44; Tr 53; GP 29; GSS 8.4; MS 15.104-105; SS 13.43; L 214; GS 3.55; GK Khāta 2.

B 21U. Arithmetical mean of different lengths.
If it is irregular (distorted), the sum of the different lengths is divided by it (i.e. the number of the places measured).
When the rectangular solid to be measured has distorted surfaces, then the mean length is first obtained by:

$$\bar{a}=\frac{a_1+a_2+\ldots+a_n}{n},$$

where a_i is measured at n different places of the solid lying in the same direction.
Ex. $a=6$, $b=10$, $c_1=2$, $c_2=3$, $c_3=4$ gajas. $\bar{c}=(2+3+4)/3=3$ gajas, $V=180$ cub. gajas.

Cf. BSS 12.44; Tr 52; GSS 8.4,9-10; MS 15.105; SS 13.43; L 214; GS 3.54; GK Khāta 1.

X 22. Volume of a cylinder
Of a timber, a stone, a pillar, or a well, the volume is the same. In that case, the area of the circular field [of its base] multiplied by its thickness (i.e., height), is the volume.

$$V=Ah,$$

where A and h are, respectively, the area of the circular base (*vṛttaphala*) and the thickness (*piṇḍa*) of a pillar (*stambha*).
A. Śambhunātha gives no examples.
B.
Ex. $d=3$, $h=20$ gajas, $c=9\ 1/2$ gajas, $A=7\ 1/8$ sq. gajas $=7;\ 3$ sq. gajas, $V=142\ 1/2$ cub. gajas.
Cf. Tr 57; L 214; AP 47.26; GS 3.60.

X 23. Volumes of a sphere and a cone-like heap of grain

Half the cube of the diameter of a sphere is increased by one eighteenth [of itself]. [The result is the volume of the sphere.] The square of one sixth of the circumference, multiplied by the height, is the volume (of a cone-like pile of grain].

$$V=\frac{d^3}{2}+\frac{d^3}{2}\cdot\frac{1}{18}\left[=\frac{19}{6}\cdot\frac{d^3}{6}\right] \qquad \text{(X 23ab).}$$

where d is the diameter (*vyāsa*) of a sphere.

$$V = (\frac{c}{6})^2 . h \qquad \text{(X 23 cd)}$$

where c and h are, respectively, the circumference (*vṛtta*) of the circular base and the height (*udaya*) of a cone-like heap of grain.

A. Śambhunātha mistakenly regards the latter half of the verse as giving another rule for the volume of a sphere, and replaces in his commentary "the height" (*udaya*) by "the diameter" (*vyāsa*). (Cf. GS. 3.65 and Uddeśapañcaka 25, where Pherū too employs the word *udaya* for the diameter of a sphere.) According to him, therefore, the second rule is:

$$V = (\frac{c}{6})^2 . d \; [= (\frac{\pi}{6})^2 d^3] \qquad \text{(X 23cd)},$$

where d and c are, respectively, the diameter and the circumference of a great circle of a sphere. He then refers to Bhāskara II's formula ("Līlāvatī māhiṃ tehano ekavīsamo bhāga joḍavyoṃ cheṃ"):

$$V = \frac{d^3}{2} + \frac{d^3}{2} \cdot \frac{1}{21} \; [= \frac{22}{7} \cdot \frac{d^3}{6}] \qquad \text{(Līlāvatī 201)}$$

B.
Ex.1. A sphere. $d = 3$ gajas. $V = 14\ ¼$ cub. gajas.
Ex.2. Grain. $c = 18$, $h = 2$ gajas. $V = 18$ cub. gajas.

When the grain is piled up against a wall, $V' = V - V/2 = 9$ cub. gajas.
Against the inside of a corner, $V' = V - 3V/4 = 4\ ½$ cub. gajas.
Against the outside of a corner, $V' = V - V/4 = 13\ ½$ cub. gajas.
Incidentally, Śambhudāsa refers to the weights of two kinds of grains:
1 cub. gaja of *godhūma* weighs 16 maṇas.
1 cub. gaja of *āricārī* weighs 15 maṇas.

For the same formula as (X 23ab), cf. Tr 56; MS 15.108; SS 13.46. For other formulas for the volume of a sphere, cf. Abh 2.7; Bhāskara I on Abh 2.7; GSS 8.28; L 201; GS 3.65, Uddeśapañcaka 25; GK 4a (khāta).6. For (X 23cd), cf. BSS 12.50; Tr 61-62; GP 30; MS 15.115; SS 13.50; GK Rāśi 10. See also (A 29-31) below.

A 29-31. *Grain heaped up on the ground*

The depth will be one eleventh [of the circumference of the base] if [the grain is] fine, one tenth if coarse, and one ninth if bearded. One sixth of the circumference, squared and multiplied by the depth, will be the quantity [of the grain] in cubic *hastas*. The [partial] circumference at the middle, inside, or outside is multiplied by two, four, or one and a third [severally]. The volume [obtained according to the above rule], divided by its own divisor according to whether [the mound of grain is piled] against the middle of a wall, or the inside or outside of a [rectangular] corner, is the [corrected] volume, which is regarded as being in cubic *karas* (i.e., *hastas*).

$$V = \left(\frac{c}{6}\right)^2 . h \qquad \text{(cub. hastas).}$$

The height is calculated from the circumference of its base thus:

$$h = c/a,$$

where a is 11, or 10, or 9, according to whether the grain is fine (*aṇu*), or coarse (*anaṇu*), or bearded (*śūkin*). According to Śambhunātha, fine grains are *cīṇo*, *kāṃga*, *bājaro*, and *nāgalī*; coarse grains are *gahūṃ* (?), *jūāri*, *maga*, and *amaḍa*; and bearded grains are *caṇyā*, *colā*, and *lāṃgi*.

Cf. BSS 12.50; SS 13.51; L 227; GS 3.96-97. See also (X 23) above.

If a mound of grain is piled up against the flat side of a wall, or against the inside or outside of a rectangular corner, then the circumference of the complemented base is calculated by:

$$c = \beta c',$$

where c' is the circumference by actual measurement, and β is 2, or 4, or 4/3, as the case may be. To this value (c) is applied the above rule, and the volume (V) of the complemented cone is obtained. Then, the correct volume is obtained by:

$$V' = V/\beta.$$

Śambhunātha gives no examples.

Cf. BSS 12.51; Tr 64; MS 15.117; SS 13.52; L 229; GS 3.98-100; GK Rāśi 11.

A 32. *Work for sawing a timber with a triangular or trapezoidal section*

Half of the product of the side and the upright is regarded as the area of a triangular field. Half of the sum of the thickness at the foot and top, multiplied by the length, is the area produced on the timber.

This rule gives the areas of triangular and trapezoidal sections of timbers cut by a saw.

$$A = \frac{ab}{2} \qquad \text{for the triangular section (A 32 ab),}$$

where a and b are, respectively, the side (*bāhu*) and the upright (*koṭi*) of a rectangular triangle.

$$A = \frac{a+b}{2} . h \qquad \text{for the trapezoidal section (A 32cd),}$$

where a and b are the thicknesses (*piṇḍa*) or the timber measured respectively at its foot (*mūla*) and top (*agra*), and h its length. Here also Śambhunātha refers to the relationship, A sq. aṅgulas $= A/576$ sq. hastas. He gives no examples.

For (A 32cd), cf. Tr 59-60; GP 29; MS 15.112; SS 13.48; L 223; GK Krakaca 8. For the sawing of timbers, see also BSS 12.48-49; GSS 8.63-67; GS 3.87-95.

X 24. *Telling time from a shadow*

The shadow is increased by seven. One should subtract the noon-shadow from it. Half the length of daylight, multiplied by seven and divided by it (i.e.,the result obtained above), is [the time] in *ghaṭikā*, etc.

When d is the length of daylight (*dina*) on a certain day, s_n the length of the noon-shadow (*madhyacchāyā*), and s the length of the shadow at any optional moment of the day, then the time (in *nāḍīs* or *ghaṭikās*), elapsed from sunrise if in the forenoon or remaining until sunset if in the afternoon, is obtained by:

$$t = \frac{7 . (d/2)}{s+7-s_n} \qquad \text{ghatikās (nāḍīs in B).}$$

60 ghaṭikās = 60 nāḍīs = 1 day. This is a function of three variables (s, s_n, d), and it belongs to one of the two types of approximation formulas used in ancient and medieval India. *Yavanajātaka* 79.32 and *Pañcasiddhāntikā* 4.48, for example, give the formula:

$$t = \frac{12(d/2)}{s-s_n+12} \qquad \text{nāḍīs.}$$

Cf. GSS 9.15.

These two formulas can be expressed by a single formula.

$$t \frac{g(d/2)}{s-s_n+g},$$

where g is the height of the gnomon, which is usually taken to be 12 *aṅgulas* (fingers) in India. A gnomon of 7 units has not been attested yet in any other Sanskrit works, but a gnomon of "7 feet" is known to have been used in a Syriac work written in A.D. 1279, in which a gnomon of "12 fingers" is also mentioned [Neugebauer 1975, 744].

A formula of the other type has been given by Brahmagupta (BSS 12.52):

$$t = \frac{d/2}{s/g \pm 1},$$

which is equivalent to the former type when $s_n = 0$.

Cf. Tr 65; GP 30; GSS 9.8; GK Chāyā 12. Cf. also [Davidian 1960], [Pingree 1970, 121-122], and [Stone 1982].

A. Śambhunātha does not refer to the method for obtaining the length of daylight (d); he simply gives the maximum and the minimum values:

max. = 33;34 ghaṭikās,

min. = 27;34 ghaṭikās.

This combination is impossible since the sum of the maximum and the minimum should equal 60 *ghaṭikās*. As for the noon-shadow (s_n), he prescribes a curious method.

The longest noon-shadow = max. daylight − min. daylight
= 33;34 − 27;34 = 6.

The next value = 6 − 6/6 = 5.

The rest is unknown since he stops here.

If, following Śambhunātha, we assume the difference between the max. and the min. to be 6, then we have:

max. = 33 ghaṭikās,

min. = 27 ghaṭikās,

since their sum is 60. This ratio, 33;27, corresponds to the geographical latitude, $\phi \simeq$ 21;28 degrees.

He also gives a versified table of twelve noon-shadows (A 1°), which are gross (*sthūla*) according to him.

B. Śambhudāsa's recension does not indicate explicitly that the result of the computation, $s + 7 - s_n$, is the divisor of the division prescribed. He gives one verse each for the length of daylight (B 25) and the noon-shadow (B 26).

Ex. On the day of the spring equinox. When does the gnomon-shadow reach eleven [units]?

Given: $s = 11$.

Solution: d = 30 ghaṭīs, from (B 25).
s_n = 3, from (B 26).

$$t = \frac{7 \cdot (30/2)}{11 + 7 - 3} = 7 \text{ ghaṭīs.}$$

Therefore, it is 7 ghaṭīs after sunrise, or before sunset.

A 1°. Length of the noon-shadow.

The [length of the] noon-shadow is: two, one, zero, ("a shadow" in ms.), one,[one and a half], two and a half, four less a quarter, five, six, five, five less a quarter, and three and a half.

The ms. being corrupted, this is only a tentative translation based on the text restored with the help of the table in the ms., which is also corrupted:

s_n = 2, 1, 0, 1, 2½, 2½, 3¾, 5, 6, 5, 4¾, 3½.

When compared with Śambhudāsa's a table of noon-shadows (B 26), these values show the following peculiarities, provided that the entry is assumed, as usual, to be the beginning of each zodiacal sign.

1) The integer values, 7 in number, form a linear zig-zag function (dotted lines in Fig.1), but it has been slid to the left by one zodiacal sign for some reason unknown to us.

2) The fractional values, 5 in number, deviate from that linear function. The deviations, as they stand now, are not consistent, but, intentionally or not, every fraction has become closer to Śambhudāsa's corresponding value.

B 25. Length of daylight

The [number of] days [elapsed] from the first point of Capricorn, multiplied by three, increased by zero-three-five-one, and divided by sixty. The quotient is [the length of daylight] in *kalā*. From Cancer [is obtained the length of] the night in the same way.

When n is the number of days elapsed from the beginning of Capricorn (*markarādi*) or from the beginning of Cancer (*karkādi*), then the length of the day or of the night, respectively, on that day is calculated by:

$$d = \frac{3n + 1530}{60} \qquad \text{(kalās)}$$

1 kalā = 1 nāḍī = 1 ghaṭikā. This formula yields 34;30 and 25;30 *ghaṭikās* respectively as the lengths of the longest and the shortest days. This ratio, 34;30:25;30 = 23:17, corresponds to the geographical latitude, ϕ≈30;24 degrees. Compare this with the latitude of Ahmedabad, ϕ = 23;03 degrees, where Śambhudāsa wrote this commentary.

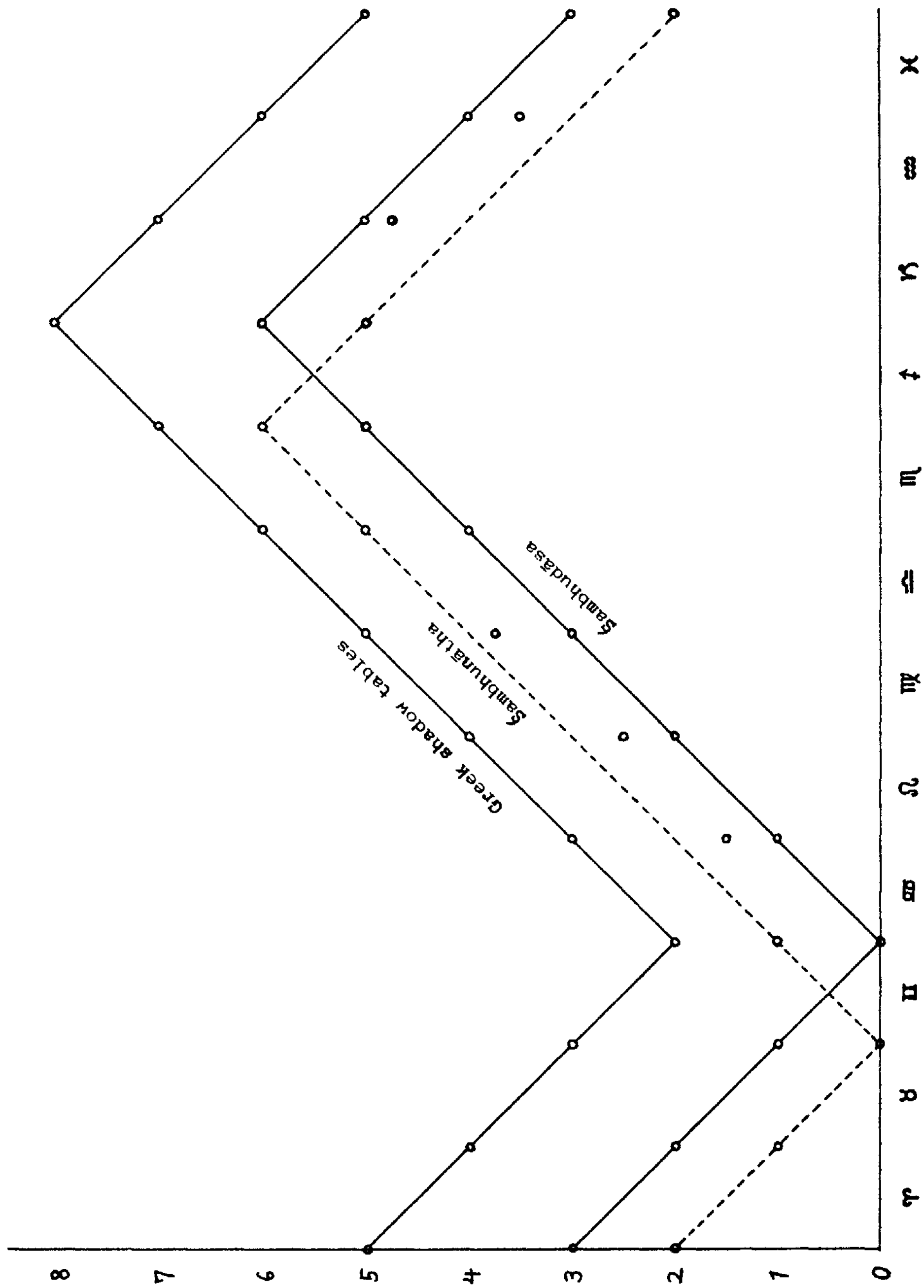


Fig. 1. Noon-shadow

B 26. Length of the noon-shadow

The noon-shadows have been declared in order to be three, two, one, zero, one, two, three, four, five, six, five, and four, from the [time of the sun's] entrance into Aries.

This verse enumerates 12 values of the noon-shadow (s_n) assigned for the day when the sun enters each zodiacal sign beginning with Aries (*meṣa*).

$$s_n = 3, 2, 1, 0, 1, 2, 3, 4, 5, 6, 5, 4.$$

These values are used in (X 24). They satisfy a linear zig-zag function of the ordinary type (see Fig. 1). The difference between the maximum and the minimum values, 6, is unusual in India, but it is common in "the Greek shadow tables" of Byzantine codices: in the basic pattern of those tables the max. and the min. lengths of the noon-shadows are 8 and 2 "feet" (or units) respectively [Neugebauer 1975, 738]. Cf. [Yano 1986, 22-26].

[X 25. Colophon of the *Pañcaviṃśatikā* (?)]

Colophon of A (in prose).

Thus is completed the *Gaṇitasāra* composed by Śrī Śambhunātha. [This manuscript was] written on the 13th day of the dark half of the month of Māgha in Saṃ 1786 by Caturavijaya at Śrī Bhujanagara for the sake of its recital. May there be prosperity, good fortune, and welfare. Thus, good fortune will be to both the scribe and the reader.

The date given here corresponds to February 15, Wednesday, (± 1 day), A.D.1730. Bhujanagara is probably modern Bhuje in Kaccha, about 300 km west of Ahmedabad.

B 27. Colophon of B (in verse and prose).

In the year of five-eight-four-one, at the city of Ahammadāvāda, the *Bālabodhāṅkavṛtti* was composed by Śambhudāsa.

(In prose) Thus is completed the *Bālabodhāṅkavṛtti* on the *Pañcaviṃśatikā*, composed by the wise (or the minister) Śambhudāsa.

The year 1485 given here is, as in the colophon of A. no doubt that of the Saṃvat era, and corresponds to A.D. 1428/29 .

APPENDICES

Notation:

A = Śambhunātha's commentary.

B = Śambhudāsa's commentary.

APPENDIX 1. MULTIPLICATION TABLE

	In A 12 & 15	In B 17.
2 × 2 = 4:	dū dū cyāra.	—
3 × 3 = 9:	traṇa trī nava.	—
4 × 2 = 8:	cyāra dū āṭha.	cyāri dū āṭha.
4 × 4 = 16:	cyāra co kūṃ solaṃ.	cyāri cu kū sola.
9 × 3 = 27:	nava trī sattāvīsa.	—
10 × 10 = 100:	dāhe dohe so.	—

APPENDIX 2. WEIGHTS AND MEASURES

Weights

A.

tola for gold.
dhara(?) for baladī(?).
māṣa for gold.
sera for ghee.

B.

gadīyāṇa for gold and rūpa (silver?).
caḍatu for "100-caḍatu ṭaṅka" and "60-caḍatu ṭaṅka".
tola for gold.
maṇa for rice, mudga beans, godhūma, āricāri, mañjīṭha, dānta (ivory?), ṣāṇḍa, sūkadi, ācchī, paḍasūtra, seī for seeds.

According to Śambhudāsa (in B)

one cub. gaja of godhūma weighs 16 maṇas,
one cub, gaja of āricāri weighs 15 maṇas.

Money

A.

korī.
dokaḍā.
28 dokaḍās = 1 korī.

This korī is most probably a copper coin minted by Sultan Muzaffar Shah III (1562-72). See [Sircar 1968, 23].

B.

jathala (= jital).
jayasthala (= jital).
ṭaṅka.
dramma.
3 jathalas = 1 dramma(?)
48 jayasthalas = 1 ṭaṅka.

This ṭaṅka is a silver coin issued by Muḥammad bin Tughluq (1325-51), which is equivalent to 48 jitals. See [Sircar 1968, 20].

Length

A.

aṅgula.

hasta.

24 aṅgulas = 1 hasta.

B.

jyeṣṭha aṅgula.

gaja.

visā.

24 jyeṣṭha angulas = 1 gaja.

20 visās = 1 gaja.

Time

A.

ghaṭikā.

B.

kalā.

Appendix 3. Prices

A.

One sera of ghee costs 8 dokaḍās.

A woman 16 years old costs 125 korīs.

One horse costs 235 korīs in one example, and 355 ½ korīs in another.

The investments of three persons are 51, 68, and 85 korīs.

B.

A woman of 16 years old costs 200 ṭaṅkas.

A certain pearl costs 25 drammas of 100-caḍatu ṭaṅka.

One gadīyāṇa of rūpa (silver?) costs 18 drammas.

One tola of gold costs 32 ṭaṅkas.

One maṇa of mañjīṭha costs 35 ṭaṅkas.

One maṇa of dānta (ivory?) costs 36 ṭaṅkas.

One maṇa of ṣāṇḍa costs 81 drammas.

One maṇa of sūkadi costs 64 ṭaṅkas.

One maṇa of ācchī costs 188 drammas.

One maṇa of paḍasūtra costs 308 ṭaṅkas.

The investments of three persons are 100, 500, and 1000 ṭaṅkas.

ABBREVIATIONS

Abh = *Āryabhaṭīya* of Āryabhaṭa I, ed by K. S. Shukla.
AP = *Aparājitapṛcchā* of Bhuvanadeva, ed by P.A. Mankad.
BG = *Bījagaṇita* of Bhāskara, ed by V.G. Āpaṭe.
BGA = *Bījagaṇitāvataṃsa* of Nārāyaṇa, ed by K.S. Shukla.
BM = *Bakhshālī Manuscript*, ed by T. Hayashi.
BSS = *Brāhmasphuṭasiddhānta* of Brahmagupta, ed by S. Dvivedin.
GK = *Gaṇitakaumudī* of Nārāyaṇa, ed by P. Dvivedin.
GM = *Gaṇitamañjarī* of Gaṇeśa, India Office Library, Eggeling 2881.
GP = *Gaṇitapañcaviṃśī* of Śrīdhara, ed by D. Pingree.
GS = *Gaṇitasāra* of Ṭhakkura Pherū, ed by Agaracanda and B. Nāhaṭā.
GSS = *Gaṇitasārasaṃgraha* of Mahāvīra, ed by M. Raṅgācārya (also ed by L. C. Jain)
GT = *Gaṇitatilaka* of Śrīpati, ed by H.R.Kāpadīa.
L = *Līlāvatī* of Bhāskara II, ed by V. G. Āpaṭe (also ed by K.V. Sarma).
MS = *Mahāsiddhānta* of Āryabhaṭa II, ed by S. Dvivedin.
PG = *Pāṭīgaṇita* of Śrīdhara, ed by K.S.Shukla.
SS = *Siddhāntaśekhara* of Śrīpati, ed by B.Misra.
Tr = *Triśatikā* of Śrīdhara, ed by S.Dvivedin.

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ERRATUM

Article: The *Pañcaviṃśatika* in Its Two Recensions – Takao Hayashi, IJHS, 26 (4), 1991, pp. 395-448

Page	Line	Instead of	Read
400	6	('[the ... [verses])	("[the ... [verses]")
	9	another unnumbered	another, unnumbered
401	4	*Pātīganita* 14	*Pāṭīgaṇita* 14
	-2	verses	verse
404	15	*Ganitapañcavimśī* "Mathematics in twenty-five *ślokas*" "ascribed to Srīdhara	*Gaṇitapañcaviṃśī* ("Mathematics in twenty-five [*ślokas*]") ascribed to Śrīdhara
	20	*Ganitapañcavimśī*	*Gaṇitapañcaviṃśī*
	23	*Lilāvatī*	*Līlāvatī*
	24	*Lilāvatī*	*Līlāvatī*
	-9	Lunkāgaccha	Luṅkāgaccha
	-8	*Iṣṭānkapañcavimśatikā*	*Iṣṭāṅkapañcaviṃśatikā*
	-7	numbers",	numbers"),
	-6	*śārdūlavikriḍita*	*śārdūlavikrīḍita*
405	4	*nirvāna*	*nirvāṇa*
	5	number 470	the number 470
	-15	*Sambhudāsa*	*Śambhudāsa*
	-12	in	is
407	4	moon-shadow	noon-shadow
	13	*Iṣṭānka*	*Iṣṭāṅka*
	-12	Antyaṃ	antyaṃ
	-7	X 2)	(X 2)
408	3	vyavakalitaṃ munibhiḥ	vyavakalitaṃ kathitaṃ munibhiḥ
	8	dyiguṇī	dviguṇī
	13	is	in
	-11	mūlya-	mūlya
	-6	(corr.i.m)	(corr.i.m.)
409	17	ghajet	bhajet
	19	dviduṇana	dviguṇana
	23	pūrva-	pūrva
	-9	paraspara [ṃ]	paraspara[ṃ]
	-1	ādibhāga [ḥ]	ādibhāga[ḥ]
410	6	dyayau	dvayau
	21	phalam	phalaṃ
	26	saṃyuthaḥ	saṃyutaḥ
	28	vyāsattriguñitā	vyāsattriguṇitā
	-5	dviguṇā-	dviguṇādviyukta
411	5	(bhujai	bhujai
	6	tryasre[2]	tryasre[3]
	22	pāṣāṇayo[ḥ]	pāṣāṇayo[ḥ][1]
	-14	hataḥ	hataḥ[3]
	-10	['n]	['ṅ]
	-5	madhyathaḥ	madhyataḥ
412	5	gatā [ḥ]	gatā[ḥ]
	12	ṣaṣ[ṭ] er	ṣaṣ[ṭ]er

Under the last line of p. 412, add the following:

Translation with Mathematical Commentary

All the examples and their solutions given below are those found in the commentaries, unless otherwise indicated.

X 1. Benediction

A. Śambhunātha's recension reads:

Having paid homage to Mahādeva ("Great God", Śiva) first, I shall tell [you] this [work named] *Pañcaviṃśatikā* with my own *sūtras* (i.e., versified rules) in order to enhance the intellect of biginners.

B. Śambhudāsa's recension reads "with five *sūtras* (i.e., sections)" instead of "with my own *sūtras*." See the Introduction above.

413	12	*pararḋha*	*parārdha*
	-15	(i.e., next]	(i.e., next)
414	3	*Sn*	S_n
	6	*Sn*	S_n
	11	*Sn*	S_n
	13	(*pāda*), of	(*pāda*) of
	-10	$S_{10}(10 + 1)$	$S_{10} = (10 + 1)$
415	3	*samkalita*	*saṃkalita*
	6	*Sn*	S_n
	-5	$\overline{nm}$	$n\bar{m}$
416	13	increased	decreased
	-3	*Kapātasandhi*	*Kapāṭasandhi*
	-3	A cow's	a cow's
418	9-10	8 7 4 8 2	8 7 4 8 2
419	-12	a 10	^{a}10
	-13	b 5321	^{b}5321
420	-5	(12 x 3) (12 x 3)	(12 x 1) (12 x 3)
421	-9-10	Delete the horizontal line under "Ex. 2."	
423	15	Prakīṇaka	Prakīrṇaka
424	-2	have	halve
425	5	(o = odd, e = even) 55225	(o = odd, e = even)
	7	(5 - $\underline{2}^2$ = 1)	(5 - $\underline{2}^2$ = $\underline{1}$)
	16	$\underline{4}$ 6	4 $\underline{6}$
427	-10	Ḡss 5,2;	GSS 5.2;
428	20	a_1	a_i
	20	*M*, the profit	and *M* the profit
	22	p^i	p_i
430	8	W <	W =
	17	(w_1): ... 4 masas.	(w_i): ... 4 māṣas.
	21	becomes	become
431	1	(W_i)	(w_i)
	3	varṇnas	varṇas
	8	gadīyāna	gadīyāṇa
	17	*a and b*	*a* and *b*
	18	ablong	oblong
	-14	A Square	A square

432	-19	$\sqrt{a^2} + r$	$\sqrt{a^2 + r}$
	-15	·c. [=	c [=
433	15	vimiśite	vimiśrite
	-15	jāyaī	jāyaï
	-4	$\sqrt{10}(d/2)^4$	$\sqrt{10(d/2)^4}$
434	-5	$\sqrt{n}p^2$	$\sqrt{np^2}$
	-2	$\sqrt{n}p^2$	$\sqrt{np^2}$
435	16	AP 47.24 gives	AP 47.25 gives
	6	$\sqrt{10}[h(a+h)/2]^2/9$	$\sqrt{10[h(a+h)/2]^2/9}$
436	1	"315"	"3I5"
	3	"315"	"3I5"
	11	"315"	"3I5"
	13	"315"	"3I5"
	16	of a diameter	of the diameter
	-10	$s = a + b + c + d$.	$2s = a + b + c + d$.
437	13	later	latter
	17	GK 4.13-14	GK Kṣetra 13-14
	22	(*vijra*)	(*vajra*)
438	-17	base] multiplied	base], multiplied
439	-11	GK 4a(khāta).6	GK Khāta 6
440	5	c/a	c/α
	6	where a is	where α is
441	4	or	of
	-15	nādīs	nāḍīs
	-10	t	$t =$
443	12	a table	table
	-15	Capricorn, multiplied	Capricorn is multiplied
	-10	*markarādi*	*makarādi*
445	-6	of A. no doubt	of A., no doubt
446	21	sūkadi, ācchī, paḍasūtra, seī for seeds.	sūkadi, ācchī, and paḍasūtra. seī for seeds.
	-15	one cub, gaja	one cub. gaja
448	14	Kāpadīa	Kāpadīā
	15	Bhāskara II,	Bhāskara,
	-1	*paddhati*) *Indo-*	*paddhati*). '*Indo-*